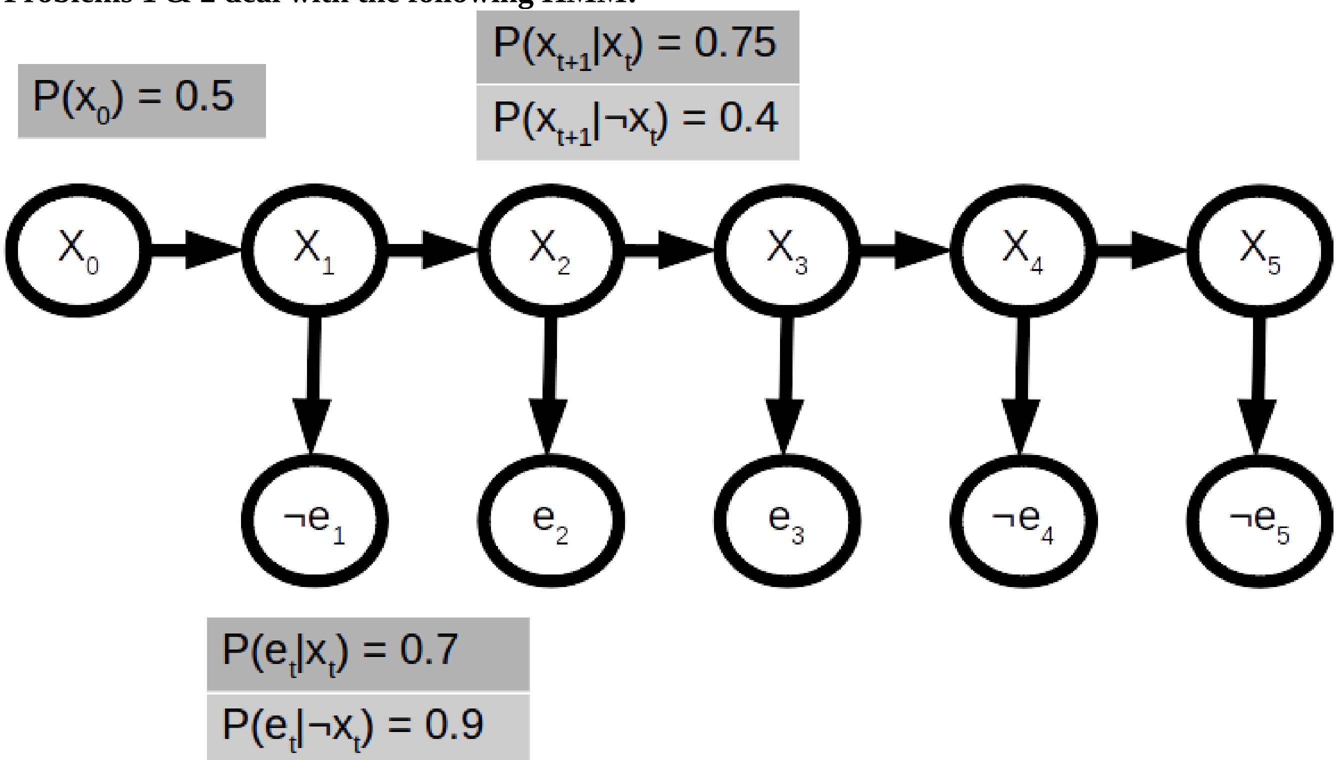
CSci5512, Fall-2021
ASSIGNMENT 3:
Assigned: 10/23/21 Due: 11/4/21 at 11:55 PM (submit via Canvas, you may scan or take a picture of your paper answers) Please organize your work before submitting.
On all problems you must show work to receive full credit; all answers done individually

Problems 1 & 2 deal with the following HMM:



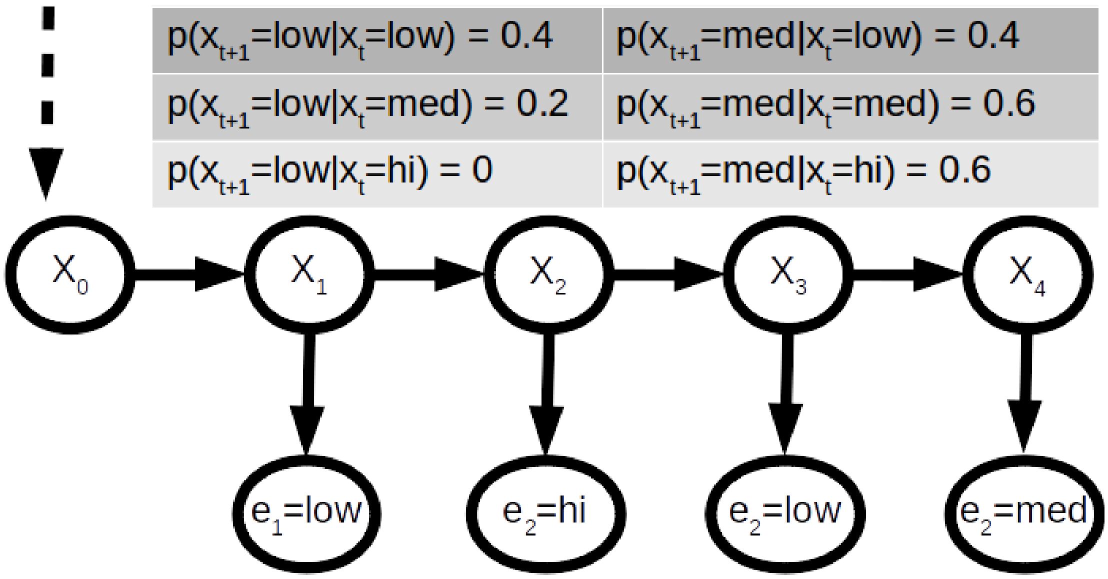
Problem 1. (15 points)
Write a program to compute the smoothed values on the Hidden Markov-Model (HMM). Give answers to:
(1) What is the forward (un-normalized) message for each of the five days.
(2) What is the backward (un-normalized) message for each fo the five days.
(3) What is the final smoothed probabilities (normalized) for each day. (Note: day 5's smoothed is just filtering.)

Problem 2. (10 points)
Suppose you figured out that the forward message on day 100 was <0.2, 0.5> (un-normalized). The evidence on day 100 was: e_{100}=true. What was the forward message on day 99?

Problem 3. (20 points)
Use the HMM below to find the most-likely explanation (MLE) (i.e. Verbiti algorithm) for the four days given the evidence sequence: e_1=low, e_2=high, e_3=low, e_4=med. (Note: this is the same HMM from the last homework about HMMs, though difference evidence.)

$p(x_0=low) = 0.3$	$p(x_0=med) = 0.4$

$p(x_{t+1}=low \mid x_t=low) = 0.4$	$p(x_{t+1}=med \mid x_t=low) = 0.4$
$p(x_{t+1}=low \mid x_t=med) = 0.2$	$p(x_{t+1}=med \mid x_t=med) = 0.6$
$p(x_{t+1}=low \mid x_t=hi) = 0$	$p(x_{t+1}=med \mid x_t=hi) = 0.6$

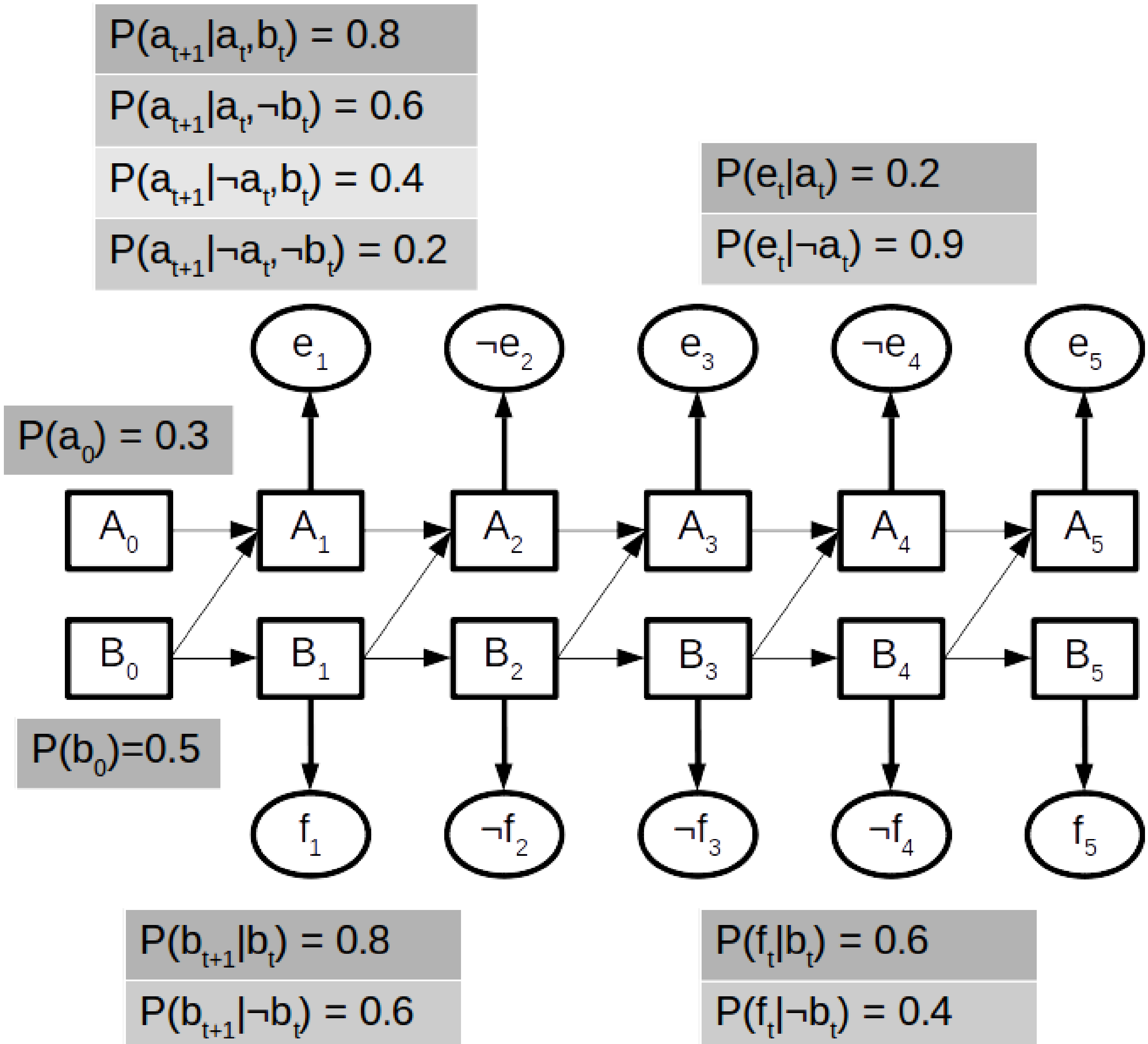


$p(e_t=low \mid x_t=low) = 0.8$	$p(e_t=med \mid x_t=low) = 0.2$
$p(e_t=low \mid x_t=med) = 0.1$	$p(e_t=med \mid x_t=med) = 0.8$
$p(e_t=low \mid x_t=hi) = 0.1$	$p(e_t=med \mid x_t=hi) = 0.4$

Problem 4. (15 points)
(1) Suppose you had a Kalman filter with $P(x_0) = N(0, 0.5)$, $P(x_{t+1}|x_t) = N(x_t, 1)$, and $P(zt|xt) = N(x_t, k^2)$. Find an approximate "k^2" such that the estimate of the variance on day 5 ($\sigma_{t=5}^2$) is 2. (Note: you probably want to use a numerical method rather than attempting to solve exactly.)

(2) Assume $k^2=2$ in the same setup from part (1). What is the mean estimate for day 5 ($\mu_{t=5}$) for the following evidence: $z_1=0.2$, $z_2=0.5$, $z_3=0.3$, $z_4=0.3$, $z_5=0.4$

Problem 5. (20 points)
Suppose we have the Bayesian network below. Use particle filtering (write a program) to estimate $P(a_5|e_1, \neg e_2, e_3, \neg e_4, e_5, f_1, \neg f_2, \neg f_3, \neg f_4, f_5)$ and $P(b_5|e_1, \neg e_2, e_3, \neg e_4, e_5, f_1, \neg f_2, \neg f_3, \neg f_4, f_5)$.



Problem 6. (20 points)
Rework your code from the previous question to use particle filtering to estimate a HMM problem with continuous variables. This will be similar to Kalman filtering, except not all the distributions will be Normal/Gaussian/Bell-curves. Specifically:
$P(x_0)$=Uniform(-1,1),
$P(x_{t+1}|x_t)$ = Uniform(x_t-1, x_t+1),
$P(z_t|x_t) = N(x_t,1)$.

Given the evidence: $z_1 = 0.5$, $z_2 = 1.4$, $z_3 = -1$... find the filtering probability distributions for days 1, 2 and 3.

Note: It is probably best to use a histogram to approximate the distributions. When resampling particles, you can either use the histogram distribution as the actual distribution or resample on top of existing particles (so some particles may duplicate/triple/etc. while others may disappear).