TAYLOR EXPANSION - 2 DIM

$f: (x, y) \longmapsto f(x, y)$ scalar valued

$$f(x+\Delta x, y+\Delta y) = f(x+\Delta x, y) + f_y(x+\Delta x, y)\,\Delta y + \frac{1}{2} f_{yy}(x+\Delta x, y)(\Delta y)^2 + O(\Delta y)^3$$

$$\begin{array}{l|ccc}
 & \parallel & \parallel & \parallel \\
 & f(x,y) & f_y(x,y)\,\Delta y & \frac{1}{2} f_{yy}(x,y)(\Delta y)^2 \\
 & + & + & + \\
f(x,y) & f_x(x,y)\,\Delta x & f_{xy}(x,y)\,\Delta x \Delta y & \frac{1}{2} f_{xyy}(x,y)\,\Delta x(\Delta y)^2 \\
 & + & + & + \\
\nabla f \circ \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix} & \frac{1}{2} f_{xx}(x,y)(\Delta x)^2 & \frac{1}{2} f_{xxy}(x,y)(\Delta x)^2 \Delta y & \frac{1}{4} f_{xxyy}(x,y)(\Delta x)^2(\Delta y)^2 \\
 & + & + & + \\
\frac{1}{2}(\Delta x\ \Delta y)\, H_f \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix} & O(\Delta x)^3 & O(\Delta x^3 \Delta y) & O(\Delta x)^3(\Delta y)^2
\end{array}$$

$$= f(x,y) + \begin{bmatrix} f_x & f_y \end{bmatrix}^* \cdot \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix} + \frac{1}{2}(\Delta x\ \Delta y)^T \begin{bmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{bmatrix}^* \begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix} + \cdots$$

$$= f(x,y) + \underline{\nabla f}_{(x,y)} \circ \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix} + \frac{1}{2}(\Delta x\ \Delta y)\; H_{f\,(x,y)} \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix} + \cdots$$

If $\underline{v} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\Delta \underline{v} = \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix}$

$$f(\underline{v} + \underline{\Delta v}) = f(\underline{v}) + \nabla_f(\underline{v}) \cdot \underline{\Delta v} + \frac{1}{2}(\Delta v)^T H_f(\underline{v})\, \underline{\Delta v}$$

Notation $\quad f_x = \dfrac{\partial f}{\partial x}$, $f_{xy} = \dfrac{\partial^2 f}{\partial x \partial y}$, etc

$= f_{yx}$ under mild conditions