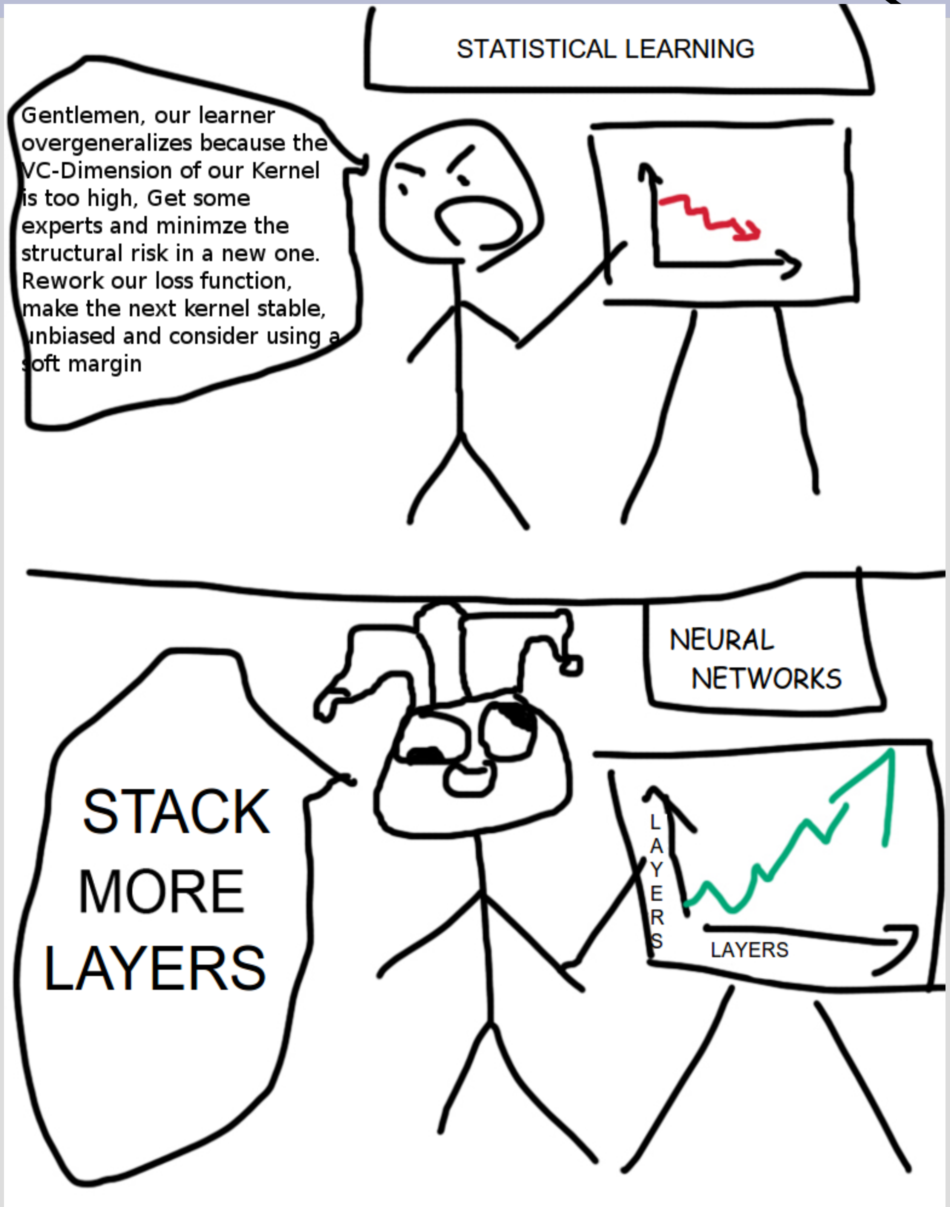
Neural networks (Ch. 12)



Back-propagation

The neural network is as good as it's structure and weights on edges

Structure we will ignore (more complex), but there is an automated way to learn weights

Whenever a NN incorrectly answer a problem, the weights play a “blame game”...

- Weights that have a big impact to the wrong answer are reduced

Back-propagation

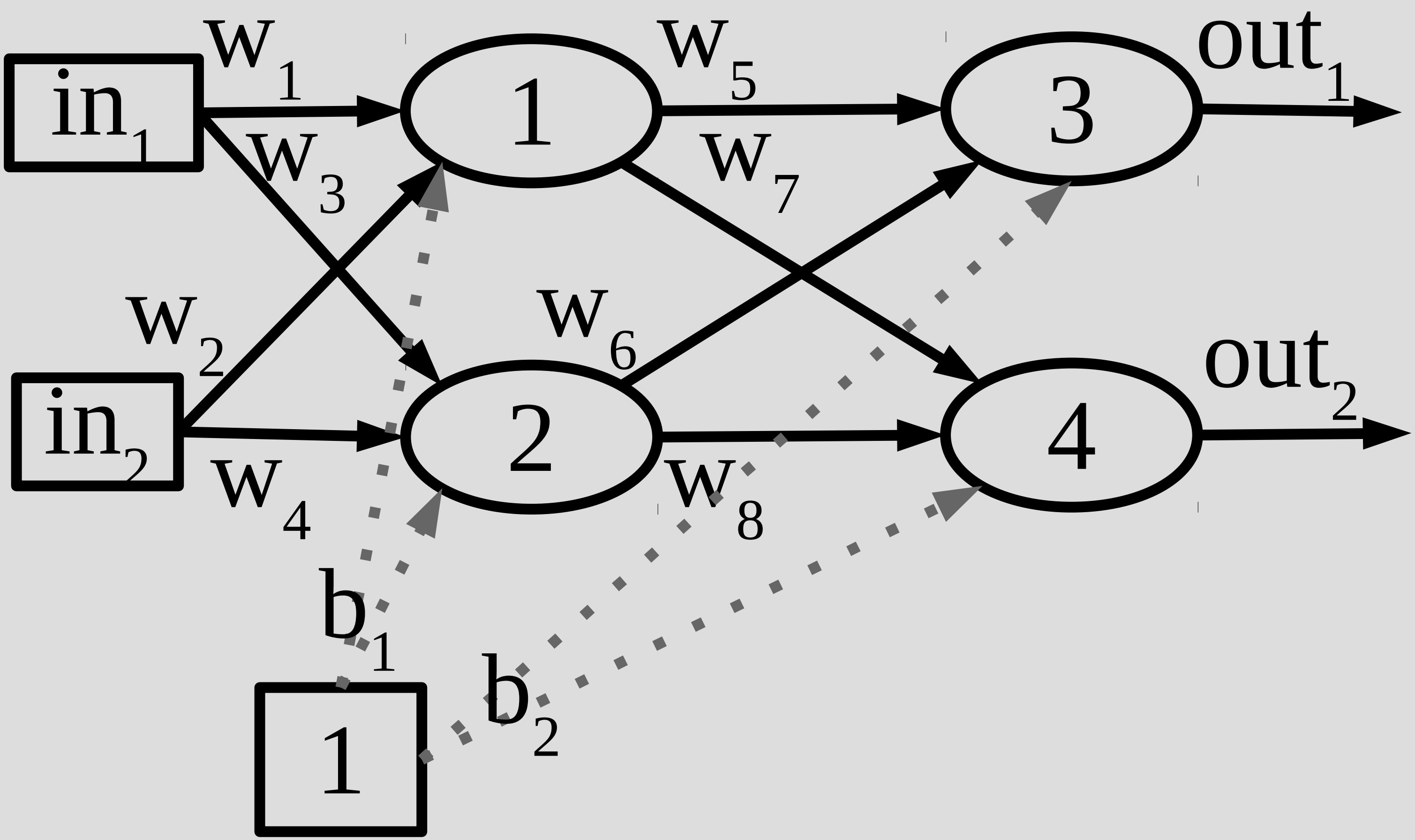
To do this blaming, we have to find how much each weight influenced the final answer

Steps:

1. Find total error
2. Find derivative of error w.r.t. weights
3. Penalize each weight by an amount proportional to this derivative

Back-propagation

Consider this example: 4 nodes, 2 layers

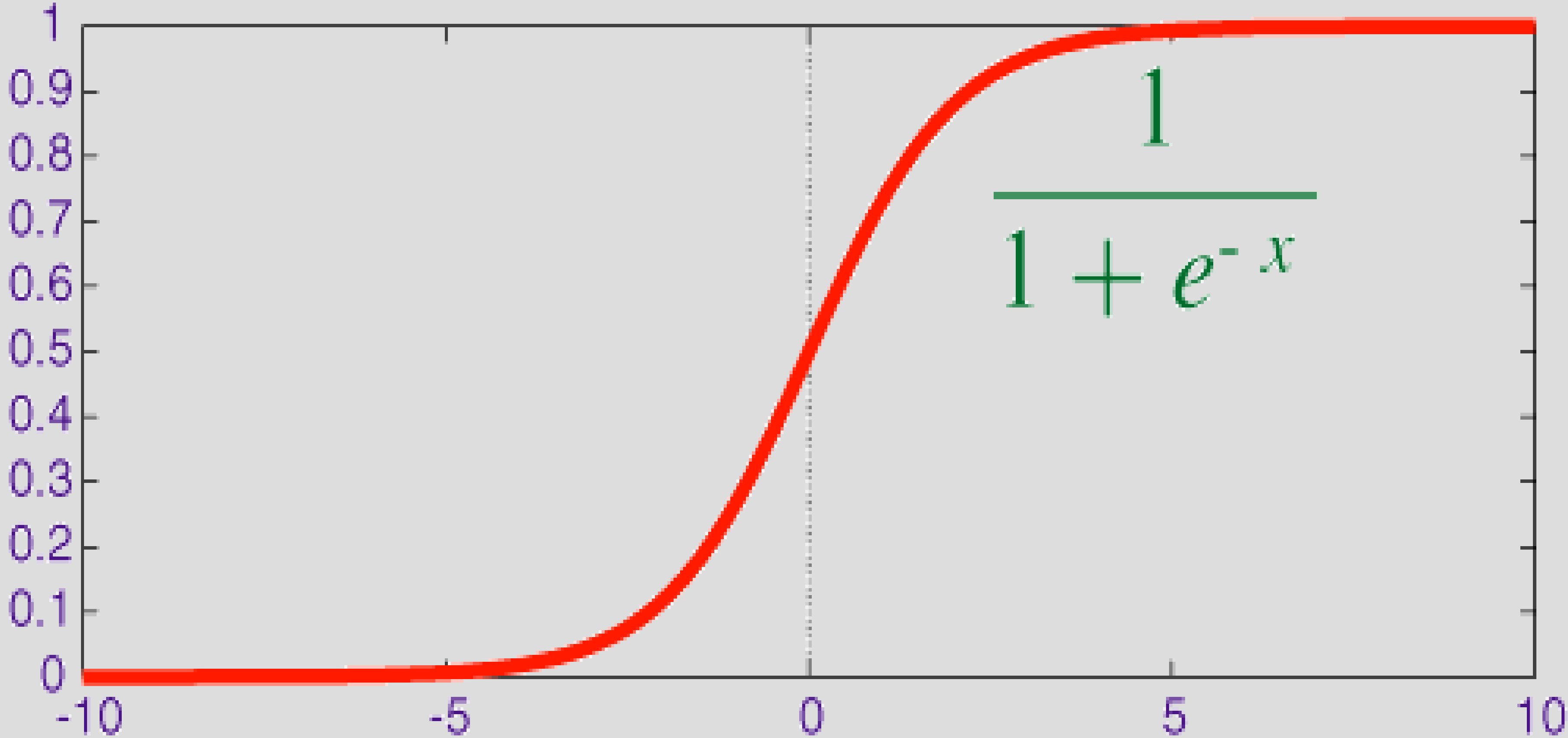


This node as a constant bias of 1

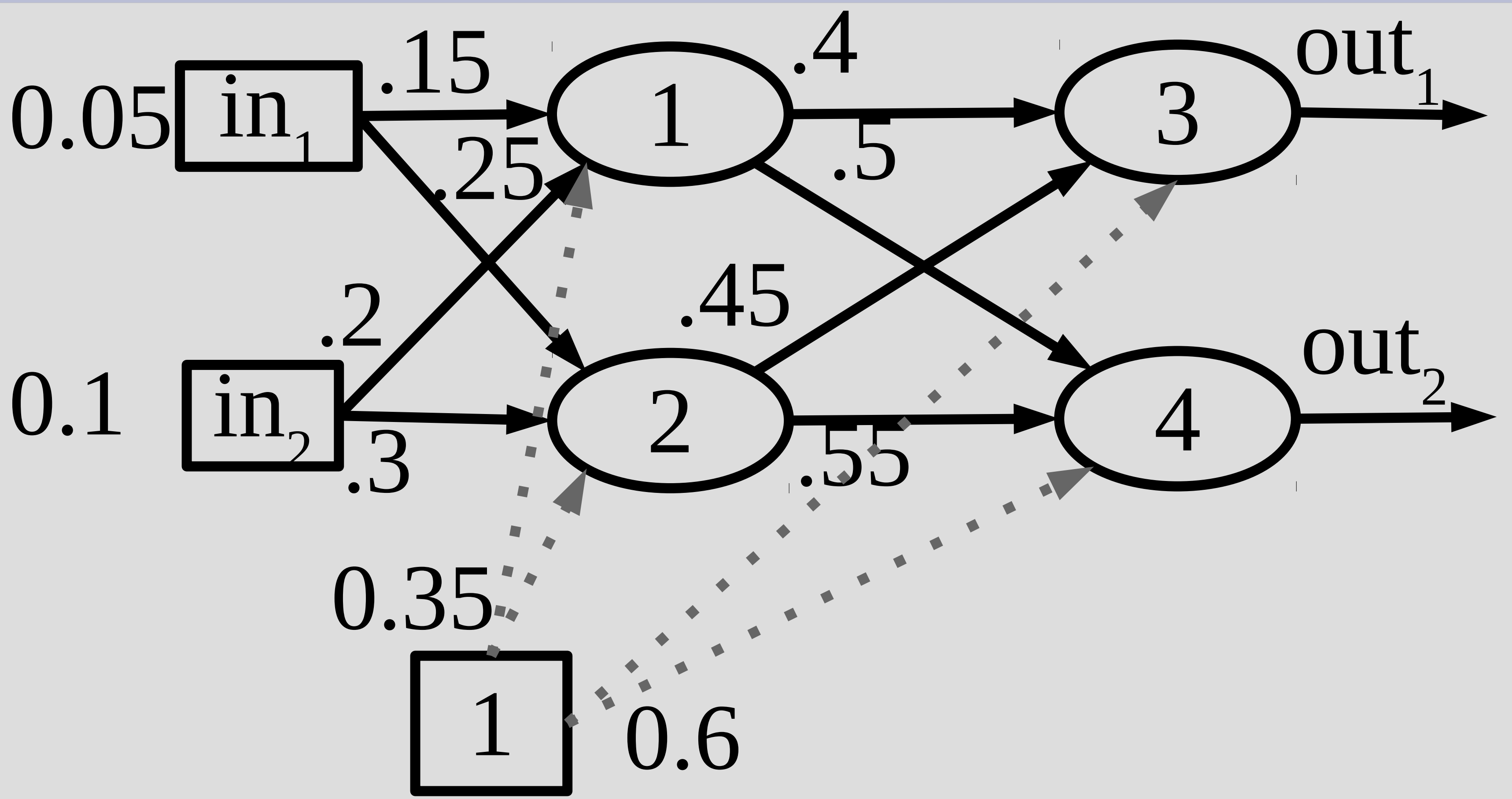
Neural network: feed-forward

One commonly used function is the sigmoid:

$$S(x) = \frac{1}{1+e^{-x}}$$

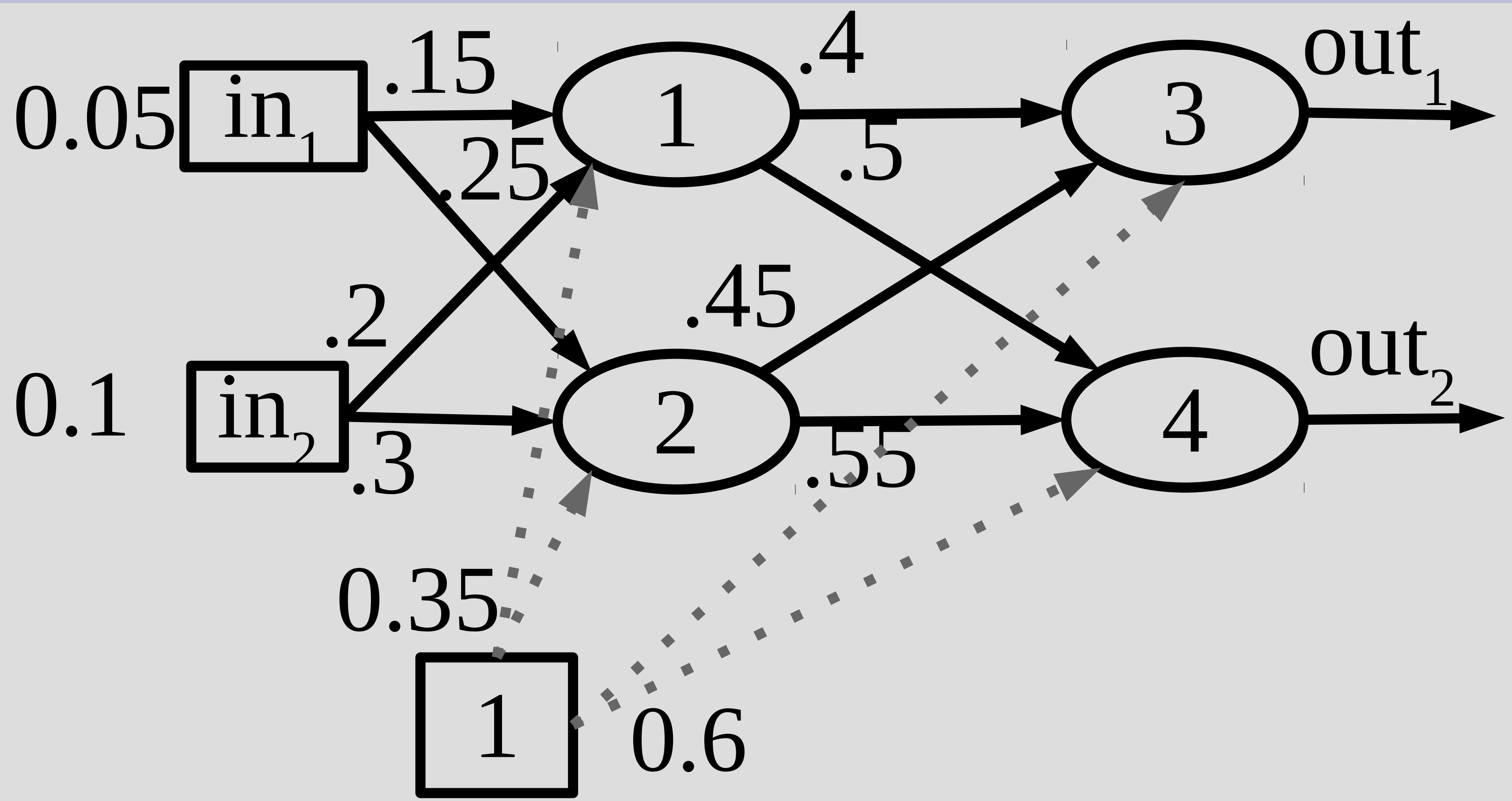


Back-propagation



Node 1: 0.15*0.05 + 0.2*0.1 +0.35 as input thus it outputs (all edges) S(0.3775)=0.59327

Back-propagation



Eventually we get: $out_1 = 0.7513$, $out_2 = 0.7729$

Suppose wanted: $out_1 = 0.01$, $out_2 = 0.99$

Back-propagation

We will define the error as: $\frac{\sum_i (correct_i - output_i)^2}{2}$
(you will see why shortly)

Suppose we want to find how much w_5 is to blame for our incorrectness

We then need to find: $\frac{\partial Error}{\partial w_5}$
Apply the chain rule:

$$\frac{\partial Error}{\partial out_1} \cdot \frac{\partial S(In(N_3))}{\partial In(N_3)} \cdot \frac{\partial In(N_3)}{\partial w_5}$$

Back-propagation

$$Error = \frac{\sum_i (correct_i - output_i)^2}{2}$$

$$\frac{\partial Error}{\partial out_1} = -(correct_1 - out_1)$$
$$= -(0.01 - 0.7513) = 0.7413$$

As $S'(x) = S(x) \cdot (1 - S(x))$

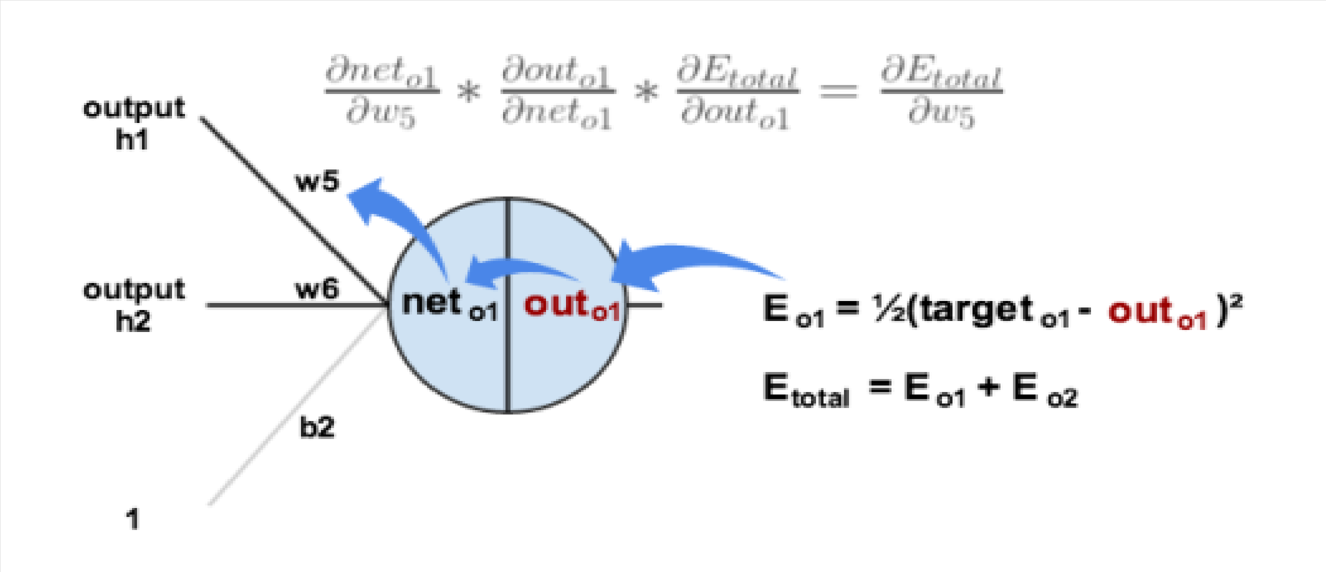
$$\frac{\partial S(In(N_3))}{\partial In(N_3)} = S(In(N_3)) \cdot (1 - S(In(N_3)))$$
$$= 0.7513 \cdot (1 - 0.7513) = 0.1868$$

$$\frac{\partial In(N_3)}{\partial w_5} = \frac{\partial w_5 \cdot Out(N_1) + w_6 \cdot Out(N_2) + b_2 \cdot 1}{\partial w_5}$$
$$= Out(N_1) = 0.5932$$

Thus, $\frac{\partial Error}{\partial w_5} = 0.7413 \cdot 0.1868 \cdot 0.5932 = 0.08217$

Back-propagation

In a picture we did this:



Now that we know w5 is 0.08217 part responsible, we update the weight by:

$w_5 \leftarrow w_5 - \alpha * 0.08217 = 0.3589$ (from 0.4)

α is learning rate, set to 0.5

Back-propagation

Updating this w_5 to w_8 gives:

$w_5 = 0.3589$

$w_6 = 0.4067$

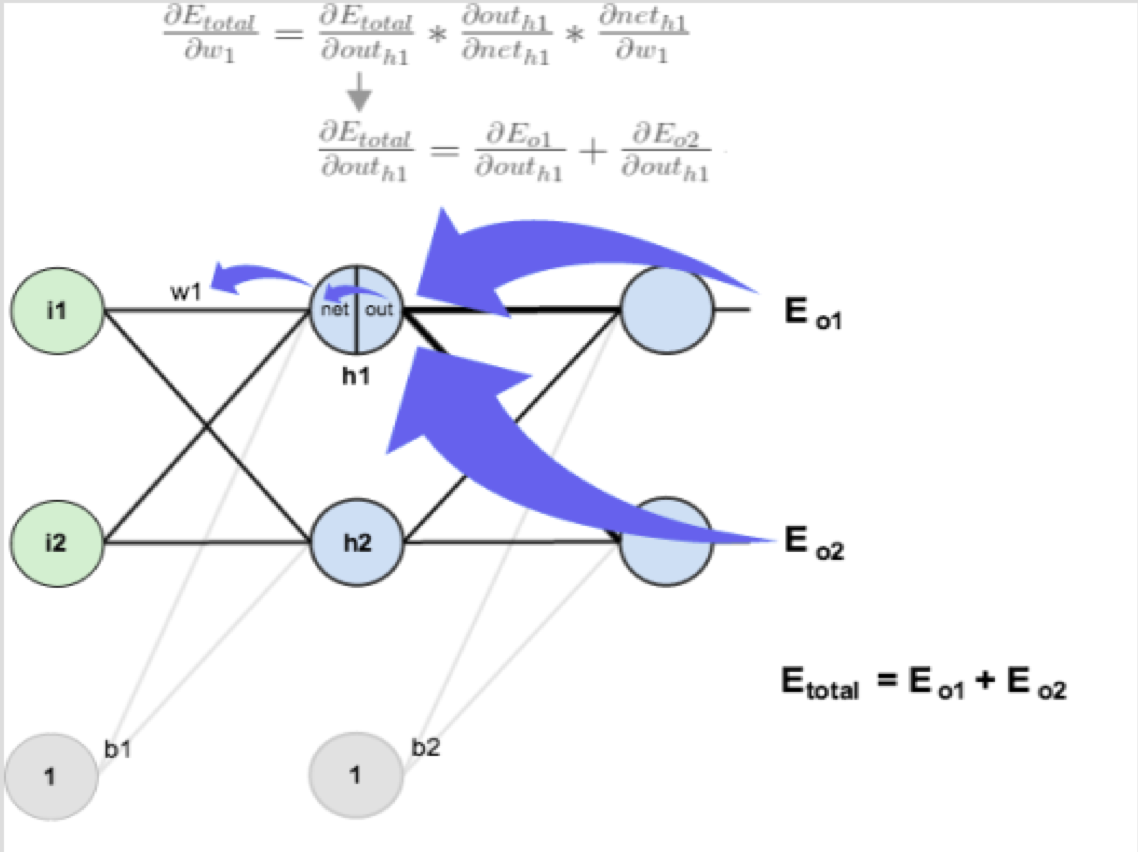
$w_7 = 0.5113$

$w_8 = 0.5614$

For other weights, you need to consider all possible ways in which they contribute

Back-propagation

For w_1 it would look like:



(book describes how to dynamic program this)

Back-propagation

Specifically for w_1 you would get:

$$\frac{\partial Error}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_1))} + \frac{\partial Error_2}{\partial S(In(N_1))}$$

$$\frac{\partial S(In(N_1))}{\partial In(N_1)} = S(In(N_1)) \cdot (1 - S(In(N_1)))$$
$$= 0.5933 \cdot (1 - 0.5933) = 0.2413$$

$$\frac{\partial In(N_3)}{\partial w_5} = \frac{\partial w_1 \cdot In_1 + w_2 \cdot In_2 + b_1 \cdot 1}{\partial w_5}$$
$$= In_1 = 0.05$$

Next we have to break down the top equation...

Back-propagation

$$\frac{\partial Error}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_1))} + \frac{\partial Error_2}{\partial S(In(N_1))}$$

$$\frac{\partial Error_1}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_3))} \cdot \frac{\partial S(In(N_3))}{\partial In(N_3)} \cdot \frac{\partial In(N_3)}{\partial S(In(N_1))}$$

From before... $\frac{\partial Error_1}{\partial S(In(N_3))} \cdot \frac{\partial S(In(N_3))}{\partial In(N_3)}$
$= 0.7414 \cdot 0.1868 = 0.1385$

$$\frac{\partial In(N_3)}{\partial S(In(N_1))} = \frac{\partial w_5 \cdot S(In(N_1)) + w_6 \cdot S(In(N_2)) + b_1 \cdot 1}{\partial S(In(N_1)}$$
$= w_5 = 0.4$

Thus, $\frac{\partial Error_1}{\partial S(In(N_1))} = 0.1385 \cdot 0.4 = 0.05540$

Back-propagation

Similarly for Error_2 we get:

$$\frac{\partial Error}{\partial S(In(N_1))} = \frac{\partial Error_1}{\partial S(In(N_1))} + \frac{\partial Error_2}{\partial S(In(N_1))}$$
$$= 0.05540 + -0.01905 = 0.03635$$

$$\text{Thus, } \frac{\partial Error}{\partial w_1} = 0.03635 \cdot 0.2413 \cdot 0.05 = 0.0004386$$

$$\text{Update } w_1 \leftarrow w_1 - \alpha \frac{\partial Error}{\partial w_1} = 0.15 - 0.5 \cdot 0.0004386 = 0.1498$$

You might notice this is small...
This is an issue with neural networks, deeper the network the less earlier nodes update

NN examples

Despite this learning shortcoming, NN are useful in a wide range of applications:

- Reading handwriting
- Playing games
- Face detection
- Economic predictions

Neural networks can also be very powerful when combined with other techniques (genetic algorithms, search techniques, ...)

NN examples

Examples:

https://www.youtube.com/watch?v=umRdt3zGgpU

https://www.youtube.com/watch?v=qv6UVOQ0F44

https://www.youtube.com/watch?v=xcIBoPuNIiw

https://www.youtube.com/watch?v=0Str0Rdkxxo

https://www.youtube.com/watch?v=l2_CPB0uBkc

https://www.youtube.com/watch?v=0VTI1BBLydE

NN examples

AlphaGo/Zero has been in the news recently, and is also based on neural networks

AlphaGo uses Monte-Carlo tree search guided by the neural network to prune useless parts

Often limiting Monte-Carlo in a static way reduces the effectiveness, much like mid-state evaluations can limit algorithm effectiveness

NN examples

Basically, AlphaGo uses a neural network to “prune” parts for a Monte-carlo search