

**2nd Midterm Exam**  
**Thursday April 12**  
**75 minutes – Open book and notes**

1. *15 points*

Convert these English sentences to predicate calculus, using reasonable predicates such as Student, Class, etc:

1. Some students took the class French1001 in Spring 2017.
2. Only one student took French3001. (In this case you are NOT allowed to use the quantifier “ $\exists!$ ” for “there exists only one”)
3. Nobody buys a trip to the moon.
4. There is a seller who sells cars only to people who are rich.
5. A person born outside the UK, if one of their parents is a UK citizen-by-birth, is a UK citizen-by-descent.

2. *10 points*

Convert the following sentences into a form in which all the quantifiers are as far to the left as possible without changing the meaning of the sentence.

1.  $\forall x [[\exists y Loves(x, y) \vee Loves(y, x)] \Rightarrow Happy(x)]$
2.  $\forall x [Happy(x) \Rightarrow [\exists y Loves(x, y) \vee \exists z [Cat(z) \wedge Owns(x, z)]]]$

3. *15 points*

You are given the following pairs of expressions where upper case letters indicate constants, lower case letters indicate variables, functions, or predicates. Each pair is independent of the others. In each pair variables with the same name are the same variable.

For each specify if they can be unified and the results of the unification process. If they cannot be unified explain briefly why.

1.  $p(B, C, z, f(A, z))$  and  $p(y, z, C, w)$
2.  $r(f(y), y, x)$  and  $r(x, f(A), f(v))$
3.  $q(f(A, x), x)$  and  $q(f(z, f(z, D)), z)$

4. *20 points*

You are given the following sentences “Heads I win. Tails you lose. If I win, you lose. If you lose, I win”

1. Represent them in propositional calculus using the following propositions *Head*, *Tail*, *IWin*, *YouLose*.

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2. Suppose that you are told “Head.” Prove, using modus ponens or resolution with refutation, that “You lose.”
3. Suppose that instead you are told “Tail.” Can you prove that “I do not win”? If not, why not?

5. *25 points*

You are given the following knowledge base:

1. If an object is on top of another, then that object supports it.
2. If an object is above another object and they are touching each other, then the first object is on top of the second.
3. A cup is above a book and it is touching the book.

Write the knowledge base in predicate calculus, convert to CNF, and prove by using resolution with refutation that “a book supports a cup”.

6. *15 points*

Answer these questions explaining your reasoning briefly but precisely.

1. Would you use resolution with refutation when your knowledge base is written in predicate calculus as Horn clauses? Why? If not, what would you use instead?
2. Is it true that in predicate calculus it is possible to prove any entailed sentence using resolution with refutation? Why?
3. What does it mean for resolution to be “refutation complete”? Explain with your own words.

7. *Extra Credit 10 points*

You are given the following sentences: “Tweety is a bird. Goldie is a fish. Squiggly is a worm. Kitty is my cat. Birds like worms. Cats like fish. Cats like birds. Friends like each other. Kitty is my friend. Kitty eats everything she likes.”

Write the sentences in predicate calculus and answer the following question “what does my cat eat”? If there are multiple answers, show all of them.

You do not need to show a formal proof, just answer doing logical reasoning in English. Be precise.