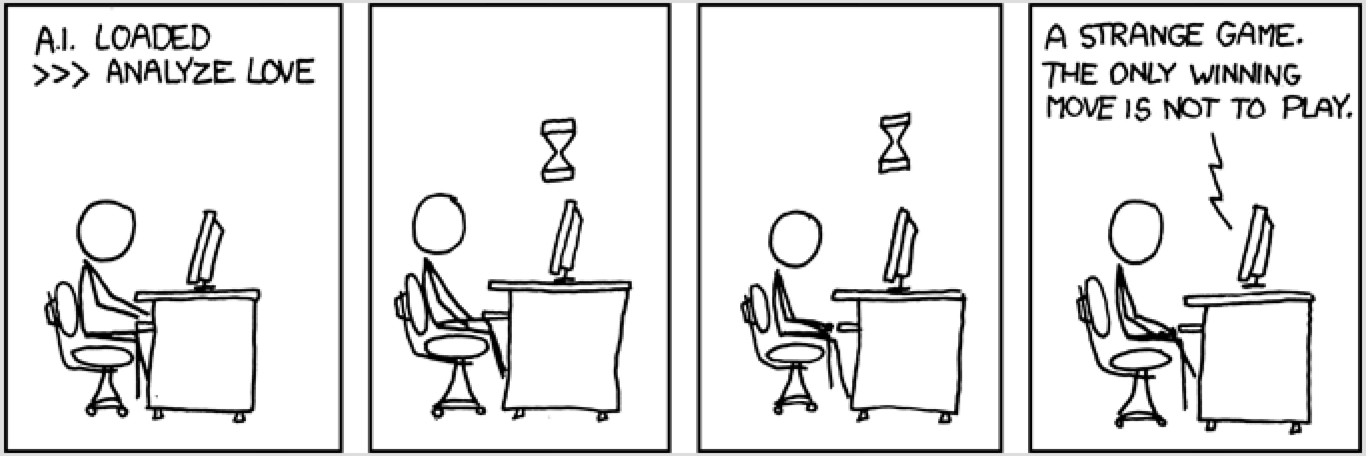
Game theory (Ch. 17.5)



Find best strategy

As a warm-up, let’s find the Nash and Pareto for this game:

3,3	0,4
3,0	1,1

Find best strategy

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3,3	0,4
3,0	1,1

Turns out there is a dominant strategy (both playing right and playing down)

So Nash is: 1,1

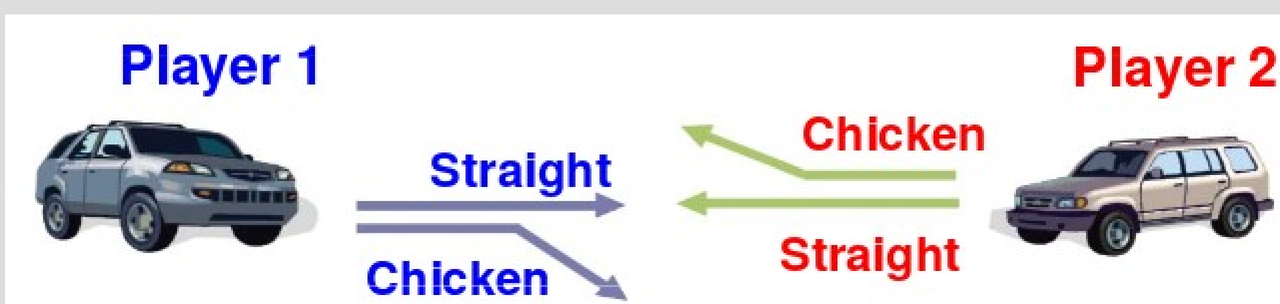
Pareto are: 3,3 and 0,4

Chicken

What is Nash for this game?
What is Pareto optimum?

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Game of Chicken



Chicken

To find Nash, assume we (blue) play S probability p, C prob 1-p

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Column 1 (red=S): p*(-10) + (1-p)*(1)
Column 2 (red=C): p*(-1) + (1-p)*(0)

Intersection: -11*p + 1 = -p, p = 1/10

Conclusion: should always go straight 1/10 and chicken 9/10 the time

Chicken

	s	c
s	-10, -10	1, -1
c	-1, 1	0, 0

We can see that 10% straight makes the opponent not care what strategy they use:

(Red numbers)

100% straight: (1/10)*(-10) + (9/10)*(1) = -0.1

100% chicken: (1/10)*(-1) + (9/10)*(0) = -0.1

50% straight: (0.5)*[(1/10)*(-10) + (9/10)*(1)]
+ (0.5)*[(1/10)*(-1) + (9/10)*(0)]
=(0.5)*[-0.1] + (0.5)*[-0.1] = -0.1

Chicken

	s	c
s	-10, -10	1, -1
c	-1, 1	0, 0

The opponent does
not care about action,
but you still do (never considered our values)

Your rewards, opponent 100% straight:

$(0.1)*(-10) + (0.9)*(-1) = -1.9$

Your rewards, opponent 100% curve:

$(0.1)*(1) + (0.9)*(0) = 0.1$

The opponent also needs to play at your value intersection to achieve Nash

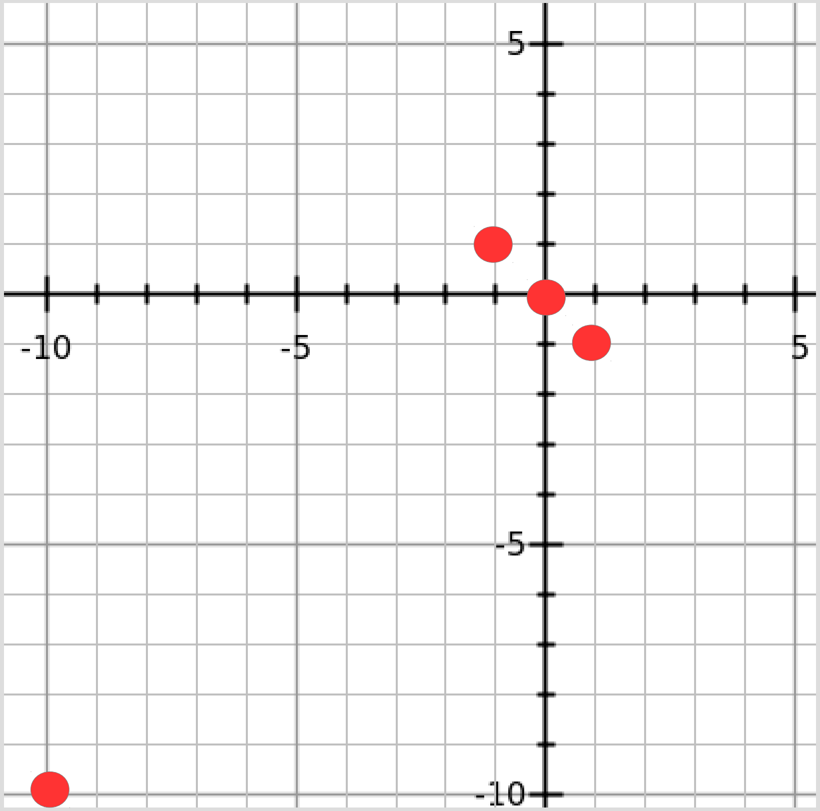
Chicken

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Pareto optimum?
All points except (-10,10)

Going off the definition, P1 loses point if move off (1,-1)
... similar P2 on (-1,1)

At (0,0) there is no point with both vals positive



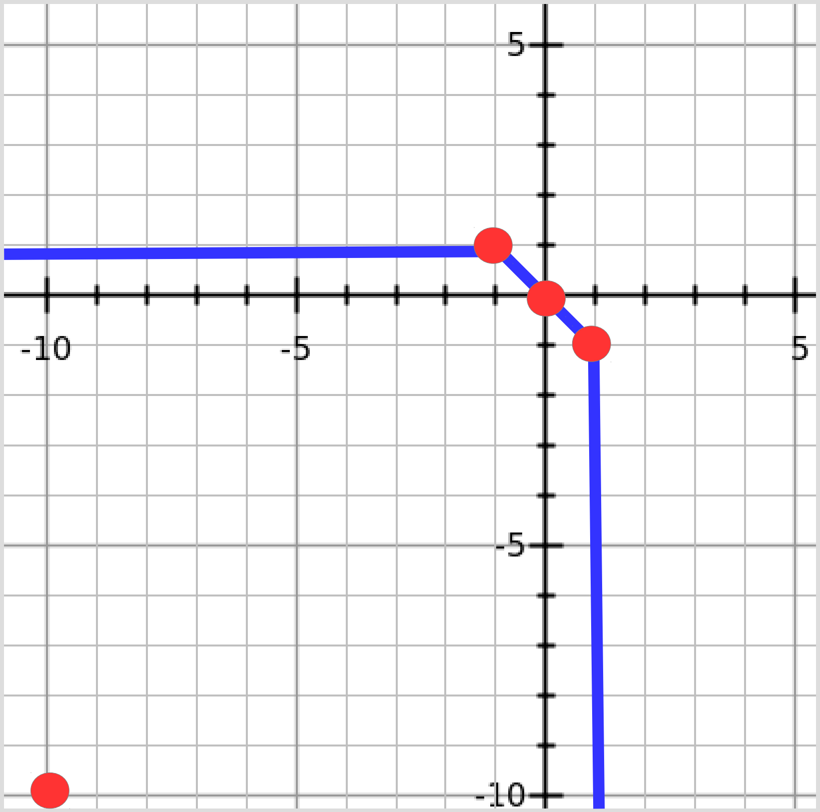
Chicken

We can define a mixed strategy Pareto optimal points

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Can think about this as taking a string from the top right and bringing the it down & left

Stop when string going straight left and down



Find best strategy

We have two actions, so one parameter (p) and thus we look for the intersections of lines

If we had 3 actions (rock-paper-scissors), we would have 2 parameters and look for the intersection of 3 planes (2D)

This can generalize to any number of actions (but not a lot of fun)

		Player 2		
		Stone	Paper	Scissors
Player 1	Stone	$(0, 0)$	$(-1, 1)$	$(1, -1)$
	Paper	$(1, -1)$	$(0, 0)$	$(-1, 1)$
	Scissors	$(-1, 1)$	$(1, -1)$	$(0, 0)$

Repeated games

In repeated games, things are complicated

For example, in the basic PD, there is no benefit to “lying”

		PRISONER 2	
		Confess	Lie
PRISONER 1	Confess	-8 , -8	0 , -10
	Lie	-10 , 0	-1 , -1

However, if you play this game multiple times, it would be beneficial to try and cooperate and stay in the [lie, lie] strategy

Repeated games

One way to do this is the <u>tit-for-tat</u> strategy:

1. Play a cooperative move first turn
2. Play the type of move the opponent last played every turn after (i.e. answer competitive moves with a competitive one)

This ensure that no strategy can "take advantage" of this and it is able to reach cooperative outcomes

Repeated games

Two “hard” topics (if you are interested) are:

1. We have been talking about how to find best responses, but it is very hard to take advantage if an opponent is playing a sub-optimal strategy

2. How to “learn” or “convince” the opponent to play cooperatively if there is an option that benefits both (yet dominated)

Repeated game

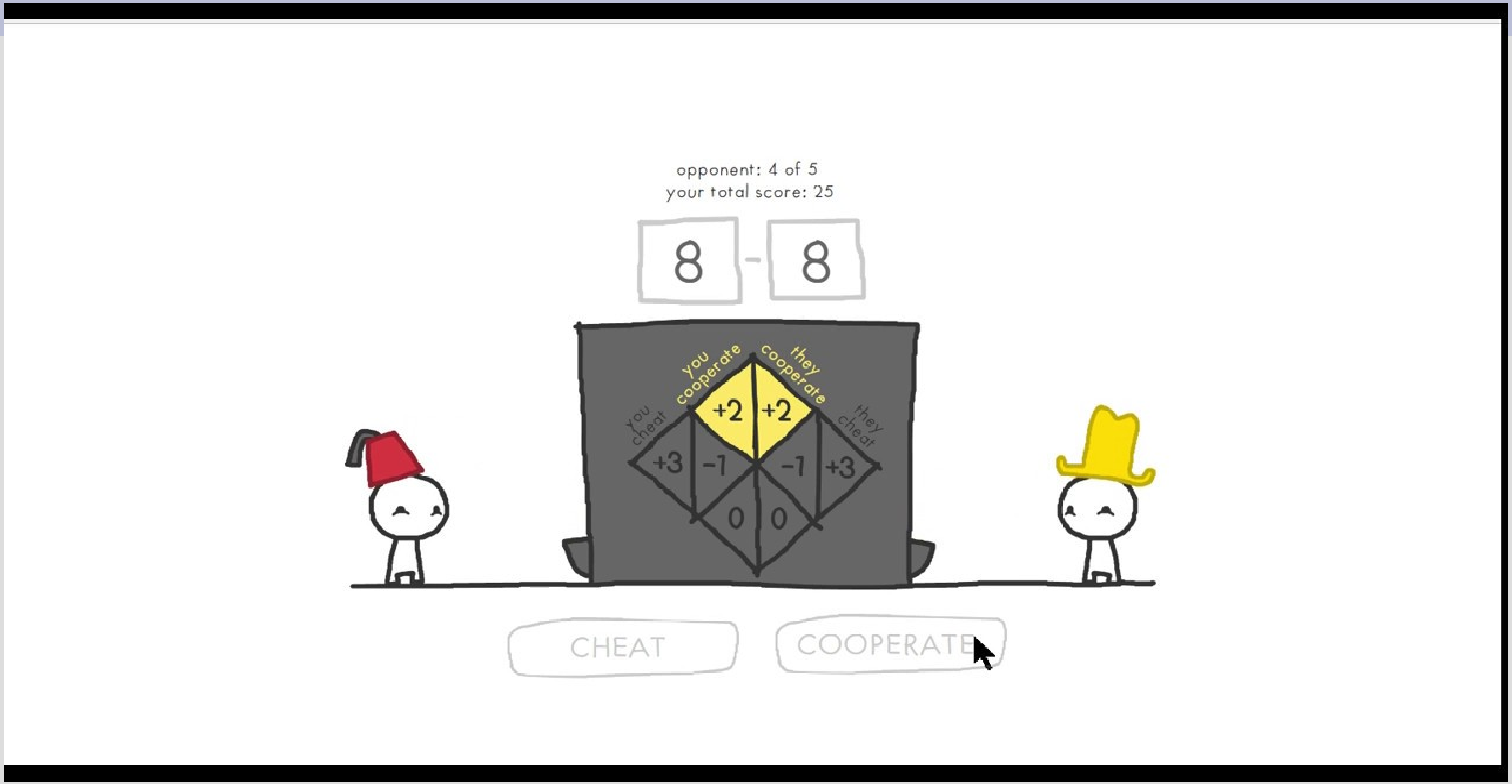
In the example from earlier... the Nash would be to play (1,1)

3,3	0,4
3,0	1,1

But, if the player cooperate, they could both achieve better results

Specifically, if player 1 flips a coin between top and bottom and player 2 chooses left ... this will average to (3, 1.5) value for them

Repeated games



http://ncase.me/trust/