

Cryptography

The goal of cryptography is to provide a "secure channel" between two (or more) parties:



- 1. Revealing no information about the messages
- 2. Delivering only messages from Ape and Bear
- 3. Delivering messages in order or not at all.







Cryptography

between two (or more) parties:

conversations.



Caesar's cipher

To encrypt a secret message,

1. Write down each plaintext letter

ATTACK AT DAWN

2. Count ahead three K letters from each plaintext letter to get a ciphertext letter.

ATTACK AT DAWN BUUBDL BU EBXO CVVCEM CV FCYP DWWDFN DW GDZQ

More keys = more security?

ATTACKATDAWN + CCCCCCCCCCC = CVVCEMCVFCYP Poly-alphabetic shift cipher:

- ATTACKATDAWN
- Rotate char 1 by K₁, rotate char 2 by K_{2r}... rotate char n by K_n, rotate char n + 1 by $K_{1,...}$
- = CTMCCDCTWCWG

- + CATCATCATCAT
- How many keys? $26^n\approx 10^{1.4n}\approx 2^{4.7n}$

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Poly-alphabetic shift cipher: Rotate char 1 by K₁, rotate char 2 by K₂,... rotate char n by K_n, rotate char n + 1 by $K_{1,r}$... How many keys? $26^n\approx 10^{1.4n}\approx 2^{4.7n}$ Is it hard to break?

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		Dely elabolistic chift sinher
	A TT A CK A TD AWN	Poly-alphabetic shift cipher:
+	CCCCCCCCCCCCC	Rotate char 1 by K1,
=	CVVCEMCVFCYP	rotate char 2 by K _{2,}
		rotate char n by K_n ,
	ATTACKATDAWN	rotate char $n + 1$ by $K_{1,r}$
+	CATCATCATCAT	How many keys? $26^n \approx 10^{1.4n} \approx 2^{4.7n}$
=	CTMCCDCTWCWG	Is it hard to break? Not if $n = K \ll M $

More keys = more security?

	Poly-al	phabetic shift cipher:
ATTACKA + CCCCCCC	CCCCCC Rotate	char 1 by K ₁ ,
= CVVCEMC	CVFCYP rotate	char 2 by K ₂ ,
	rotate	char n by K _n ,
ATTACKA	ATDAWN rotate	char $n + 1$ by $K_{1,}$
+ CATCATC	CATCAT How m	any keys? $26^n \approx 10^{1.4n} \approx 2^{4.7n}$
= CTMCCDC	CTWCWG Is it hai	rd to break? Not if $n = K \ll M $
Special cas	se when $ \mathcal{M} = \mathcal{K} $: the one-time pad

RJZJVLSSRFKNZJCTYZTSIAIWKTRSEQIYCTYISBR QIYIUKRRQEJMYIBSXYSRHZHVJSQRDPHXBSXHRAQ PHXHTJQQPDILXHARWXRQGYGUIRPQCOGWARWGQZP OGWGSIPPOCHKWGZQVWQPFXFTHQOPBNFVZQVFPYO NFVFRHOONBG JVFYPUVPOEWESGPNOAMEUYPUEOXN MEUEQGNNMAFIUEX0TU0NDVDRF0MNZLDTX0TDNWM LDTDPFMMLZEHTDWNSTNMCUCQENLMYKCSWNSCMVL KCSCOELLKYDGSCVMRSMLBTBPDMKLXJBRVMRBLUK JBRBNDKK JXCFRBULQRLKASAOCLJKWIAQULQAKTJ IAQAMCJJIWBEQATKPQKJZRZNBKIJVHZPTKPZJSI

RJZJVLSSRFKNZJCTYZTSIAIWKTRSEQIYCTYISBR QIYIUKRRQEJMYIBSXYSRHZHVJSQRDPHXBSXHRAQ PHXHT JQQPD IL XHARWXRQ GYGUI RPQ COGWARWGQZP OGWGSIPPOCHKWGZQVWQPFXFTHQOPBNFVZQVFPYO NFVFRHOONBGJVFY PUV PO EWESGPNO AMEUY PUEOXN MEUEQGNNMAFIUEXOTUONDVDRFOMNZLDTXOTDNWM LDTDPFMMLZEHTDWNSTNMCUCQENLMYKCSWNSCMVL KCSCOELLKYDGSCVMRSMLBTBPDMKLXJBRVMRBLUK JBRBNDKKJXCFRBULQRLKASAOCLJKWIAQULQAKTJ IAQAMCJJIWBEQATKPQKJZRZNBKIJVHZPTKPZJSI

RJZJVLSSRFKNZJCTYZTSIAIWKTRSEQIYCTYISBR QIYIUKRRQEJMYIBSXYSRHZHVJSQRDPHXBSXHRAQ PHXHTJQQPDILXHARWXRQGYGUIRPQCOGWARWGQZP OGWGSIPPOCHKWGZQVWQPFXFTHQOPBNFVZQVFPYO NFVFRHOONBGJVFYPUVPOEWESGPNOAMEUYPUEOXN MEUEQGNNMAFIUEXOTUONDVDRFOMNZLDTXOTDNWM LDTDPFMMLZEHTDWNSTNMCUCQENLMYKCSWNSCMVL KCSCOELLKYDGSCVMRSMLBTBPDMKLXJBRVMRBLUK JBRBNDKK JXCFRBULQRLKASAOCLJKWIAQULQAKTJ IAQAMCJJIWBEQATKPQKJZRZNBKIJVHZPTKPZJSI

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Outline

Big numbers

Modern cryptography is based on the idea of making the best (known) algorithms take too much time to be feasible.

2 ³⁰	\sim minutes on your phone
2 ⁴⁰	\sim hours on a laptop
2 ⁵⁰	\$100 on AWS
2 ⁷⁰	< 1 hour of (global) cryptocurrency mining output
2 ⁸⁰	safe for a year or two?
2 ¹²⁸	warm fuzzy cryptographer happiness zone
$2^{256} - 2^{1024}$	$\#atoms$ in the universe – $\#bit\ ops\ until\ heat\ death$



Outline

Cryptography problem statement and history

Announcements intermission

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Common cryptographic primitives

Common cryptographic primitives



Symmetric cryptography

Symmetric encryption

Comes in two forms, block ciphers and stream ciphers

Symmetric encryption

Comes in two forms, block ciphers and stream ciphers

fixed width b**l**ock ciphei

Ex: AES, 3DES, RC6 Needs to use a mode



Symmetric authentication

It's usually a mistake to use symmetric crypto that isn't also authenticated with a Message Authentication Code (MAC)

Any change to ctxt results in a new, unguessable tag.

Some block cipher modes do this automatically, e.g. AES-GCM, AES-CCM. See also ChaCha20-Poly1305



Asymmetric crypto is slower than symmetric, and requires longer keys. Normally used to share a symmetric key.



Asymmetric crypto algorithms are related to hard math (e.g., number theory) problems.

Algorithm	Problem	2 ⁸⁰ work key	2 ¹²⁸ key
RSA	factoring	1024 bits	3072 bits
Diffie-Hellman	discrete log	1024 bits	3072 bits
(Elgamal)			
ECDH	elliptic curve	160 bits	256 bits
Kyber	MLWE	~500 bytes	800 bytes
(ML-KEM)			



Signature algorithms

Signature algorithms are related to hard math (e.g., number theory) problems.

Algorithm	Problem	2 ¹²⁸ work key	Signature length
RSA	factoring	3072 bits	1024 bits
DSA	discrete log	3072 bits	320 bits
ECDSA	elliptic curve	256 bits	512 bits
EdDSA	elliptic curve	256 bits	512 bits
ML-DSA	MLWE	1312 bytes	2420 bytes

Hashing

Hash functions turn long strings into k-bit strings. Finding a preimage H(x) = y given y should take about: Finding a targeted collision $H(x_1) = H(x_2)$ given x_2 takes: Finding a free collision $H(x_1) = H(x_2)$ should take: Examples of useful hash functions: SHA256+, SHA3, BLAKE Winding down/insecure: MD5, SHA1, MD4, RIPEMD160

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Psuedorandom generators

Cryptography needs unguessable inputs.

Things not to use: serial number, Unix timestamp, process ID, hostname/address, C rand, Python random

More or less safe: _rdrand, /dev/urandom, os.random, java.security.SecureRandom