Graph Classfication: An Application of Higher Order Singular Value Decomposition

Xiaochen Zhang

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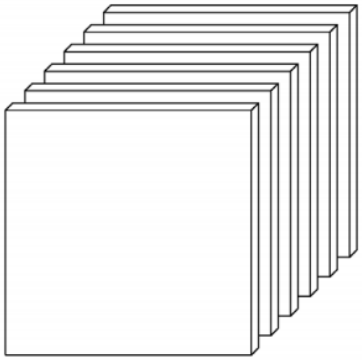
Outline

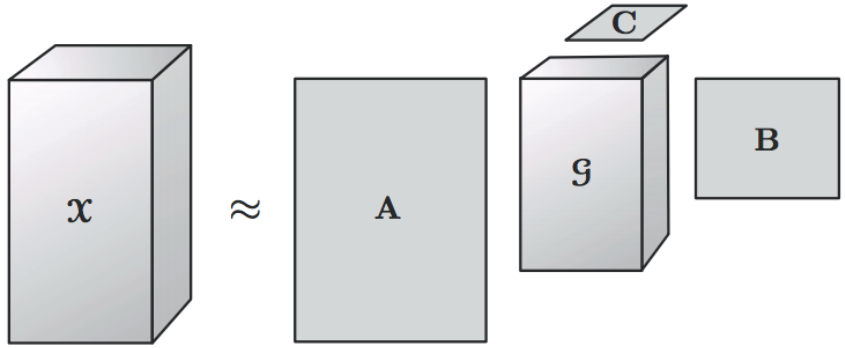
- Background
- Algorithm
- Numerical Test

Background

How to classify graphs or products?

Algorithm: Training





- Build up tensors
- Tensor decomposition: $\mathcal{X} = \mathcal{G} \times_1 A \times_2 B \times_3 C$
- Compute basis matrices: Let $D_v = \mathcal{G}(:, :, v) \times_1 A \times_2 B$, then $\mathcal{X} = \sum_{v=1}^{K} D_v \times_3 c_v$

Algorithm: Testing

- Consider optimization problem for graph x and class μ:

$$\min_{\alpha_v^\mu} ||x - \sum_{v=1}^{K} \alpha_v^\mu D_v^\mu||^2$$

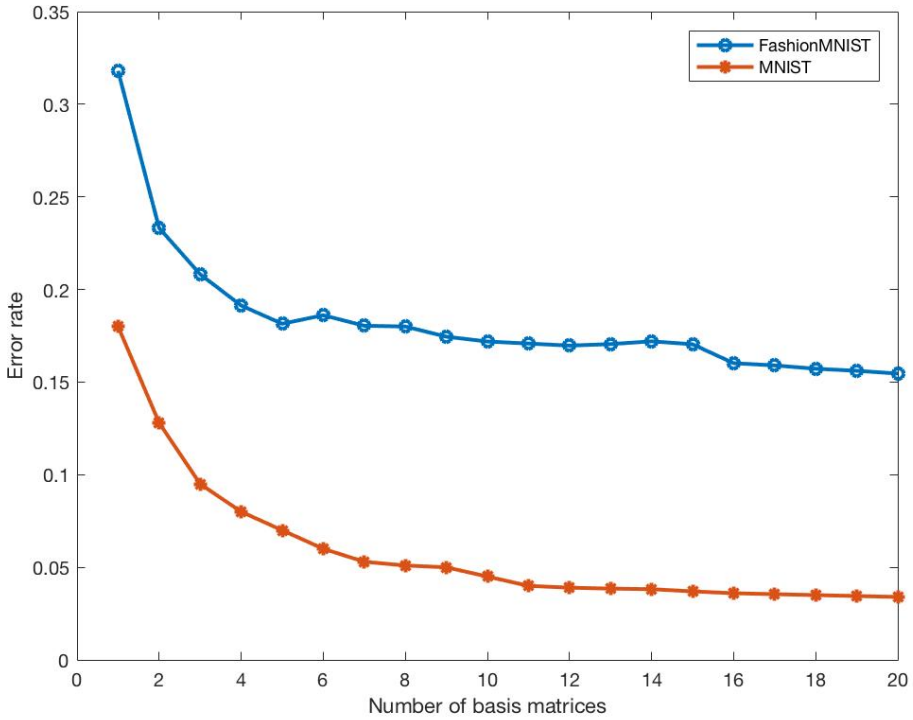
  Solution: $\alpha_v^{\mu *} =< x, D_v^\mu >$
- For each graph x and class μ, compute $R(\mu) = ||x - \sum_{v=1}^{K} \alpha_v^{\mu *} D_v^\mu||^2$
- Pick the class with smallest $R(\mu)$

Numerical Test: Settings

- Dataset: FasionMNIST (on Kaggle) and MNIST
- Size: 28×28 pixels, 60000 graphs in training set and 10000 graphs in test set
- Classes: T-shirt/top, Trouser, Pullover, Dress, Coat, Sandal, Shirt, Sneaker, Bag, Ankle boot

Numerical Test

Figure: Error rates with 1 to 20 basis matrices for FasionMNIST and MNIST



Numerical Test

Figure: First basis matrix for class 1 (Top and T-shirt)

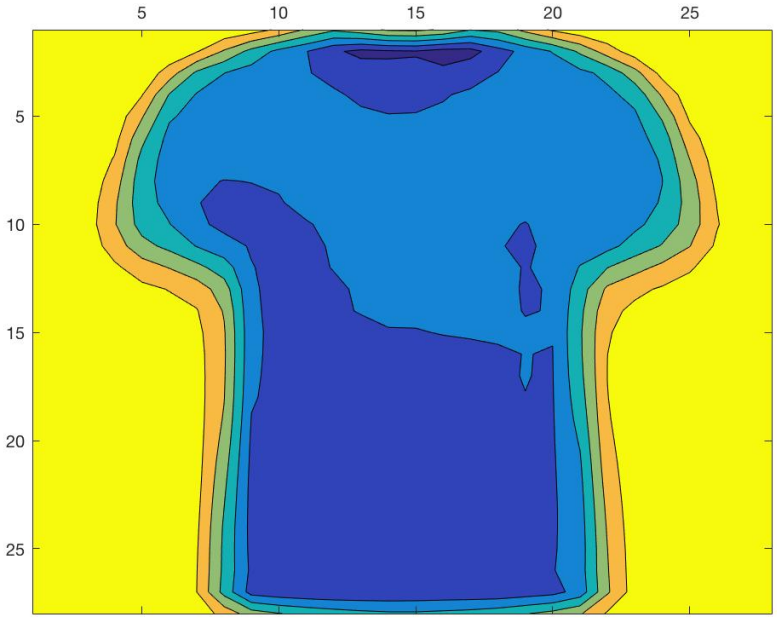
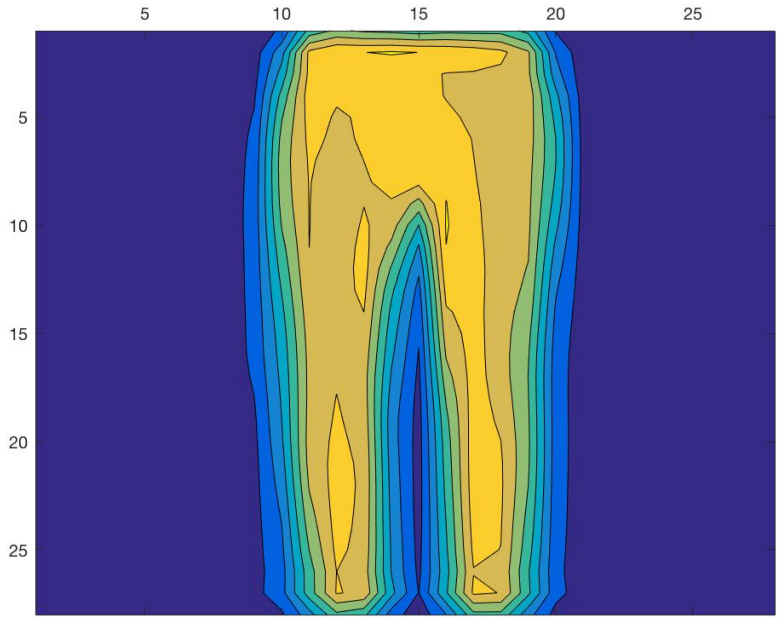
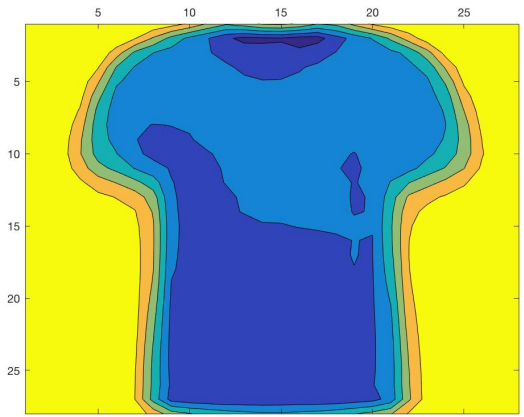


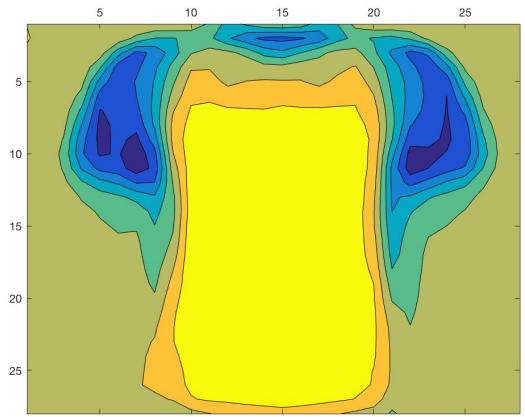
Figure: First basis matrix for class 2 (Trouser)

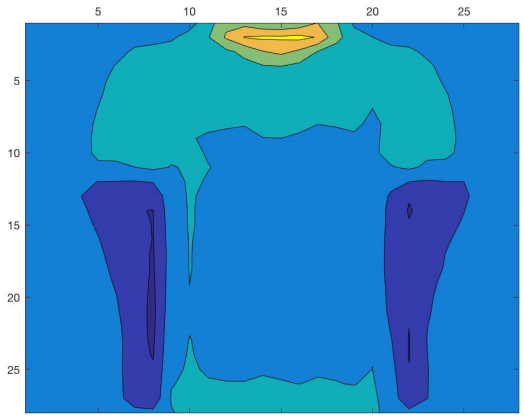


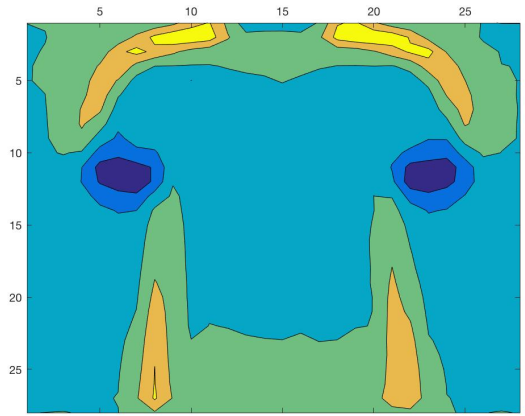
Numerical Test

Figure: Basis matrices for class 1 (Top and T-shirt)









Numerical Test

Figure: First basis matrix for class 5 (Coat)

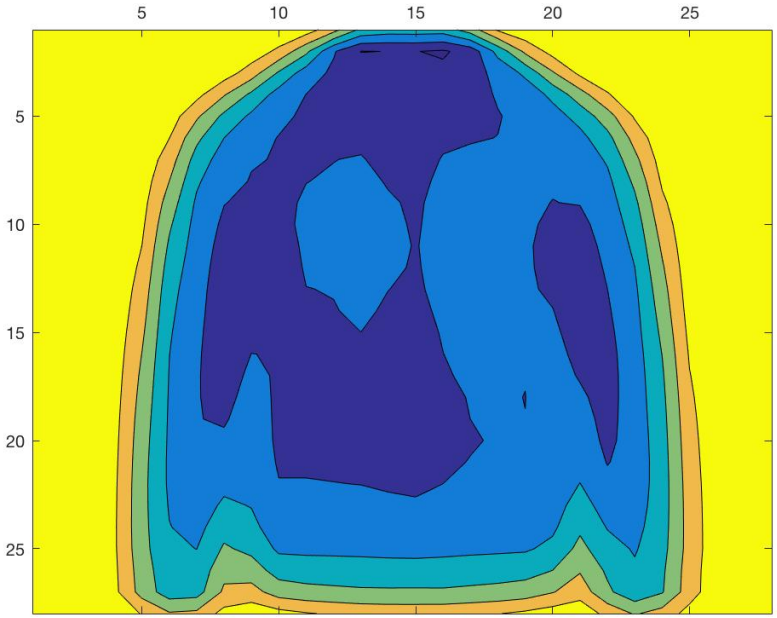
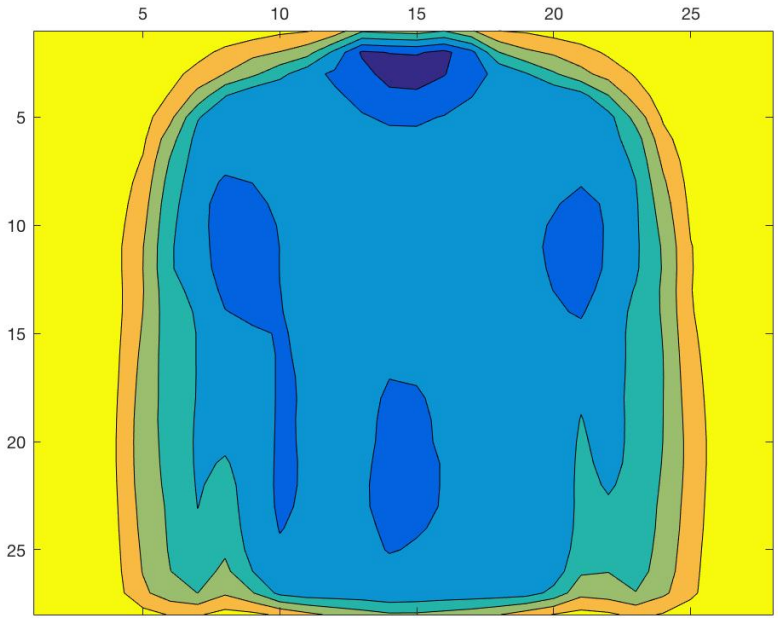
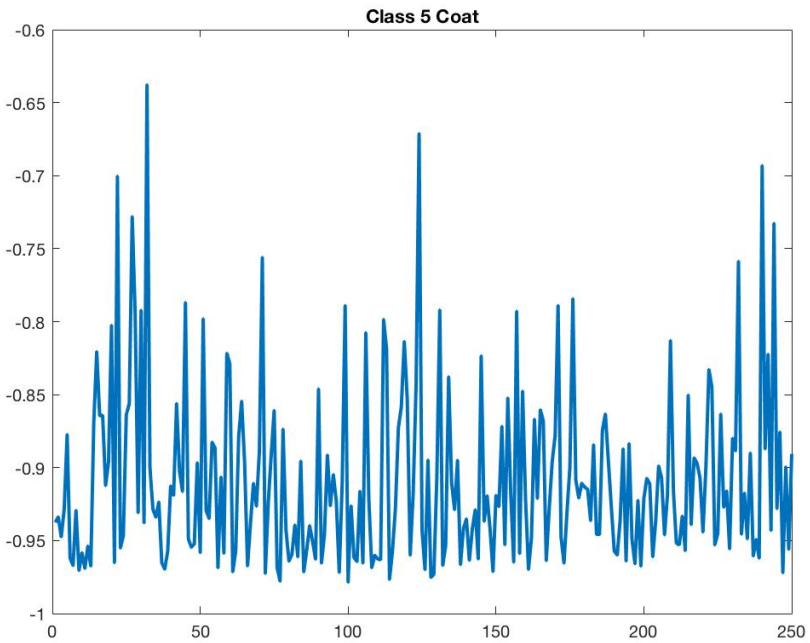


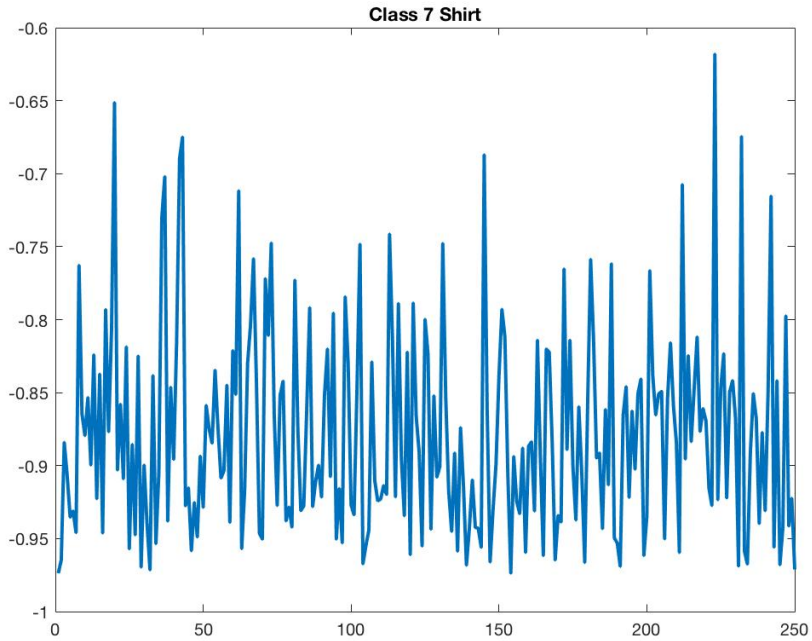
Figure: First basis matrix for class 7 (Shirt)



Numerical Test

Figure: Cosine between the first basis matrix and their respective source tensor for class 5 (Coat) and class 7 (Shirt)





Conclusion

- HOSVD classification algorithm.
- The performance of HOSVD classification on Fashion MNIST is worse than its performance on MNIST.
- Physical meaning of basis matrices.

Next Step:

- Compare with methods such as neural network.

References

[1] Savas, Berkant. "Analyses and tests of handwritten digit recognition algorithms." (2003).
[2] Savas, Berkant, and Lars Elden. "Handwritten digit classification using higher order singular value decomposition." Pattern recognition 40.3 (2007): 993-1003.

Thank you!