✍1 Show that $(I - \beta v v^T)x = \alpha e_1$ when $v = x - \alpha e_1$ and $\alpha = \pm\|x\|_2$.

Solution: Equivalent to showing that

$$x - (\beta x^T v)v = \alpha e_1 \quad \text{i.e.,} \quad x - \alpha e_1 = (\beta x^T v)v$$

but recall that $v = x - \alpha e_1$ so we need to show that

$$\beta x^T v = 1 \quad \text{i.e., that} \quad \frac{2}{\|x - \alpha e_1\|_2^2}(x^T v) = 1$$

- Denominator $= \|x\|_2^2 + \alpha^2 - 2\alpha e_1^T x = 2(\|x\|_2^2 - \alpha e_1^T x)$
- Numerator $= 2x^T v = 2x^T(x - \alpha e_1) = 2(\|x\|_2^2 - \alpha x^T e_1)$

Numerator/ Denominator = 1. □

✍2 Cost of Householder QR?

Solution: Look at the algorithm: each step works in rectangle $X(k:m, k:n)$. Step k : twice $2(m-k+1)(n-k+1)$

$$
\begin{aligned}
T(n) &= \sum_{k=1}^{n} 4(m-k+1)(n-k+1) \\
&= 4\sum_{k=1}^{n}[(m-n)+(n-k+1)](n-k+1) \\
&= 4[(m-n) * \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}] \\
&\approx (m-n) * 2n^2 + 4n^3/3 \\
&= 2mn^2 - \frac{2}{3}n^3
\end{aligned}
$$

□