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matrix-vector product - (column form)

$$\begin{vmatrix} -1 & 2 & 1 \\ 0 & 1 & 2 \\ -2 & 1 & 0 \end{vmatrix} x + \begin{vmatrix} 1 \\ -2 \\ -1 \end{vmatrix} = 1x \begin{vmatrix} -1 \\ 0 \\ -1 \end{vmatrix} - 2x \begin{vmatrix} 2 \\ 1 \\ 1 \end{vmatrix} - 1x \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix}$$

transpose conjugate:

$$A = \begin{vmatrix} 1+i & 3-i & 0 \\ 2+i & 2 & -1 \\ 1 & 5-i & 0 \end{vmatrix} \quad A^H = \begin{vmatrix} 1-i & 2-i & 1 \\ 3+i & 2 & 5+i \\ 0 & -1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} -1 & 2 & 1 \\ 0 & 1 & 2 \\ -2 & 1 & 0 \end{vmatrix} x + \begin{vmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & 2 \end{vmatrix} = ?$$

$$= \begin{vmatrix} -1 \\ 0 \\ -2 \end{vmatrix} \begin{vmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & 2 \end{vmatrix} + \begin{vmatrix} 2 \\ 1 \\ 1 \end{vmatrix} \begin{vmatrix} 1 & 2 & 0 \\ 1 & 2 & 0 \\ 2 & -3 & 2 \end{vmatrix} + \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix} \begin{vmatrix} 1 & 0 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & 2 \end{vmatrix}$$

Example : range and null space, rank, nullity of:

$$\begin{vmatrix} -1 & 1 & 0 \\ 1 & 2 & 3 \\ 1 & -2 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

rank(A) = 2

range(A) = span of first 2 columns of A

need x such that Ax = 0

x = [1 1 -1]^T

Ax = 1st col + 2nd col - 3rd col = 0

general procedure: solve $Ax = 0$ [RREF]

Null space(A) == span $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$ = set of all vector $a \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\dim(\text{Null}(A)) = 1$

$\overset{2}{\text{rank}}(A) + \overset{1}{\dim}(\text{null}(A)) = \overset{3}{3}$

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