Sep. 21 - Lecture

Prove $(Ax,y) = (x, A^H y)$

$(Ax,y) = y^H (Ax)$

$= (y^H A) x$ Note: $y^H A = (A^H y)^H$

$= (A^H y)^H x$

$= (x, A^H y)$

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for 2-norms - when Q is orthogonal then

$|| Q x || = || x ||$

Q = mxn matrix that is orthogonal: $Q^H Q = I$
Notes on terminology:
A orthogonal : columns of A are orthonormal, i.e.,
$Q^H Q = I$
mxn with m >= n

Unitary = same property when m=n.

[unitary when m = n]

$|| Qx ||^2 = (Qx,Qx) = (Q^H Q x,x) = (x,x) = ||x||^2$

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Second triangle inequality:

$| \; ||x|| - || y || \; | \le || x-y||$

$|| x || = || y + (x-y) || \le ||y|| + || x-y||$ (a)

$|| y || = || x + (y-x) || \le ||x|| + || x-y||$ (b)

(a) ==> $||x|| - ||y|| \le || x-y ||$
(b) ==> $||y|| - ||x|| \le || x-y ||$ ==> $-[||y|| - ||x||] \ge - ||x-y ||$

$-||x-y|| \le ||x|| - ||y|| \le ||x-y||$

$| \; ||x|| - || y || \; | \le || x-y||$

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