

$$A \begin{array}{ccc|c} 2 & -1 & 0 & \\ -4 & 3 & -1 & \\ 2 & -2 & 0 & \end{array}$$

$$M1 \begin{array}{ccc|c} 1 & 0 & 0 & \\ 2 & 1 & 0 & \\ -1 & & 0 & 1 \end{array} \quad M1 A \begin{array}{ccc|c} 2 & -1 & 0 & \\ 0 & 1 & -1 & \\ 0 & -1 & 0 & \end{array} \equiv A1$$

$$M2 \begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 1 & 1 & \end{array} \quad M2 A1 \begin{array}{ccc|c} 2 & -1 & 0 & \\ 0 & 1 & -1 & \\ 0 & 0 & -1 & \end{array} \equiv U$$

$$M1^{-1} = \begin{array}{ccc|c} 1 & 0 & 0 & \\ -2 & 1 & 0 & \\ 1 & 0 & 1 & \end{array} \quad M2^{-1} = \begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array}$$

$$(M1 = I - u v^T)$$

$$M1 A = A1$$

$$M2 A1 = A2 \equiv U$$

$$(M2 M1) A = M2 A1 = A2 \equiv U$$

$$A = (M1^{-1} M2^{-1}) U$$

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L

$$L = \begin{array}{ccc|c} 1 & 0 & 0 & \\ -2 & 1 & 0 & \\ 1 & -1 & 1 & \end{array}$$

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LU factorization from an algorithmic viewpoint.

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for k=1:n-1
  for i=k+1:n
    piv = A(i,k) / A(k,k) ;
    l(i,k) = piv
    A(i,:) = A(i,:) - piv*A(k,:);
  end
end

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$$piv \equiv l(i,k)$$

$$u := a(i,:)$$

step 1:

$$u := u - l(i,1) u(1,:)$$

step 2

$$u := u - l(i,2) u(2,:)$$

...

step i-1

$$u := u - l(i,i-1) u(i-1,:)$$

$$u(i,:) = a(i,:) - \sum_{k=1}^{i-1} l(i,k) u(k,:)$$

i-1

$$a(i, :) = u(i, :) + \sum_{k=1} l(i, k) u(k, :)$$

$$l(i, i) \equiv 1$$

$$a(i, :) = l(i, i) u(i, :) + \sum_{k=1}^{i-1} l(i, k) u(k, :)$$

$$a(i, j) = \sum_{k=1}^i l(i, k) u(k, j)$$

$$A = L U$$