==

Want $X \sim N(0,C)$

$C = LL^T$ Cholesky

sample: z from

$Z \sim N(0,I)$ – ``Gaussian normal"

then take:
$x = L z$

$E(x x^T) = E(L z z^T L^T) = L L^T = C$

==
Row Cholesky

from LU: (figure)

$$a(i,j) \leftarrow a(i,j) - \frac{a(k,i)}{\sqrt{a(k,k)}} \frac{a(k,j)}{\sqrt{a(k,k)}}$$

$a(i,j) \leftarrow a(i,j) - a(k,i)*a(k,j)$

==

all norms $\| . \|$ are 2-norms

$X = [x_1, x_2, x_3, \dots x_n]$
────────-
Step 1

$q_1 = x_1 / \|x_1\|$

Step 2

$\hat{q} = x_2 - (x_2, q_1) q_1$ ==> note: $(\hat{q},q_1) = 0$
$q_2 = \hat{q}/\|\hat{q}\|$

Step 3

$\hat{q} = x_3 - (x_3,q_1) q_1 - (x_3,q_2) q_2$ ==> $(\hat{q}, q_1) = 0$; $(\hat{q}, q_2) = 0$
$q_3 = \hat{q}/\|\hat{q}\|$

Step j

$\hat{q} = x_j - (x_j,q_1) q_1 - \dots - (x_j,q_{j-1}) q_{j-1}$
$q_j = \hat{q} / \| \hat{q}\|$

==
QR factorization.
In
$\hat{q} = x_j - (x_j,q_1) q_1 - \dots - (x_j,q_{j-1}) q_{j-1}$

$q_j = \hat{q} / \| \hat{q} \|$
set $r_{ij} = (x_j, q_i)$ and $r_{jj} = \| \hat{q} \|$

$\hat{q} = x_j - r_{1j} q_1 - \dots - r_{jj-1} q_{j-1}$
$r_{jj} q_j = \hat{q}$
==>
$x_j - r_{1j} q_1 - \dots - r_{j-1j} q_{j-1} = r_{jj} q_j$
------------------------ ==>

$x_j = r_{1j} q_1 \dots + r_{j-1j} q_{j-1} + r_{jj} q_j$

$$x_j = \sum_{i=1}^{j} r_{ij} q_i$$

R = upper triangular matrix n x n

write for all columns:

X = Q R

==

Modified GS

Step 3

step 1 and 2 are the same as classical

$\hat{q} = x_3 - (x_3, q_1) q_1$

$\hat{q} = \hat{q} - (\hat{q}, q_2) q_2$

$q_3 = \hat{q} / \| \hat{q} \|$

Step j

set $\hat{q} = x_j$

for i=1:j-1
    $\hat{q} := \hat{q} - (\hat{q}, q_i) q_i$ ==>makes $\hat{q} \perp q_i$
end

$q_j = \hat{q} / \| \hat{q} \|$

==

Solving the LS problem:

min $\| b - Ax \|$

Solve the normal equations:

we have A = Q R
$x = (A^T A)^{-1} (A^T b)$

$= (R^T Q^T Q R)^{-1} (A^T b)$

$= (R^T R)^{-1} (R^T Q^T b)$

$= R^{-1} R^{-T} R^T Q^T b$

$x = R^{-1} Q^T b$

Solve: $Rx = Q^T b$

====

we want : $b - Ax \perp \text{span}(A) = \text{span}(Q)$

==> $Q^T (b-Ax) = 0$

$Q^T (b-QRx) = 0$

–> $Q^T b = R x$

==

Operation cost of gram-schmidt

X : m x n

operations:

$$\sum_{j=1}^{n} [(j-1) [2m + 2m] + 3m]$$

Total cost.

$\approx 2 n^2 m$

==