===

X is m x n

$X_1 = P_1 X$

$X_2 = P_2 X_1 = P_1 P_1 X$

$X_3 = P_3 X_2 = \ldots$

$\vdots$

$X_n = P_n X_{n-1} = P_n P_{n-1} \ldots P_1 X$ = upper triangular ≡ R

R of the form [when m=7, n = 5]

$$
\begin{matrix}
x & x & x & x & x \\
0 & x & x & x & x \\
0 & 0 & x & x & x \\
0 & 0 & 0 & x & x \\
0 & 0 & 0 & 0 & x \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{matrix}
$$

$R = P_n P_{n-1} \ldots P_1 X$

$P_i^{-1} = P_i$ ==>

$[P_n P_{n-1} \ldots P_1]^{-1} = P_1^{-1} \times P_2^{-1} \ldots P_n^{-1} = P_1 P_2 \ldots P_n \equiv Q$

==>

X = Q R with $\underline{Q = P_1 P_2 \ldots P_n}$

X is m x n

Differences with Gram-Schmidt:

- here Q is of size : m x m
- R is of size : m x n – R is upper triangular.

===

How to solve LS problems?

Important : You never form Q explicitly! [m x m matrix – expensive]

A

Want to min $\underline{\underline{\| Q R x - b \|}}$ == min $\| Q^T (Q R x - b) \|$ = min $\| R x - Q^T b \|$

$R = \begin{vmatrix} R_1 \\ 0 \end{vmatrix} \quad Q^T b = c = \begin{vmatrix} c_1 \\ c_2 \end{vmatrix}$

$$
\left\| \begin{vmatrix} R_1 \\ 0 \end{vmatrix} x - \begin{vmatrix} c_1 \\ c_2 \end{vmatrix} \right\|^2 = \underline{\| R_1 x - c_1 \|^2 + \| c_2 \|^2}
$$

$$z = \begin{vmatrix} z_1 \\ z_2 \end{vmatrix} \qquad ||z||^2 = ||z_1||^2 + ||z_2||^2$$

solve $R_1 x = c_1$ ==> Done

?? How to compute $c = Q^T b$?

$Q^T = P_n P_{n-1} .... P_1$

need tp compute $Q^T b$ – done by hoApp(..., b)

==

Cost of householder:

working on a matrix of size:

(m-k+1) x (n-k+1)

matvec: 2 (m-k+1) (n-k+1)
update: 2 (m-k+1) (n-k+1)

each step 4 (m-k+1) (n-k+1)
sum from k=1 to n ——->

===========
$2mn^2 - \frac{2}{3} n^3$
===========

when m=n ==> $4/3\ n^3$

===

A =
x x x
0 x x G(1,2)
0 0 x G(1,3) G(2,3)
0 0 0 G(1,4) G(2,4) G(3,4)

===

Proofs of properties in page 9-2

Ran(P) = X ?

1) Ran(P) ⊆ X ??
Ran(P) = { z | z = P y for some y}
= { z | z = Q (Q^T y) for some y} ⊆ X

2) X ⊆ Ran(P)

x ∈ X ==> x = Q y for y ∈ $\mathbb{R}^r$
Compute Px: Px = Q Q^T (Q y) = Qy = x ==> x = Px ==>
x ∈ Ran(P) – done –