Informed Search (Ch. 3.5-3.6)
Informed search

In uninformed search, we only had the node information (parent, children, cost of actions).

Now we will assume there is some additional information, we will call a heuristic that estimates the distance to the goal.

Previously, we had no idea how close we were to goal, simply how far we had gone already.
Greedy best-first search

To introduce heuristics, let us look at the tree version of greedy best-first search.

This search will work like uniform-cost search, but use the heuristic rather than edge costs.

Specifically:
Pick minimum value from queue and add children.
Greedy best-first search

This finds the path: Arad -> Sibiu -> Fagaras -> Bucharest

However, this greedy approach is not optimal, as that is the path: Arad -> Sibiu -> Rimmicu Vilcea -> Pitesti -> Bucharest

In fact, it is not guaranteed to converge (if a path reaches a dead-end, it will loop infinitely)
We can combine the distance traveled and the estimate to the goal, which is called A* (a star)

The method goes: (red is for “graphs”)
initialize explored={}, fringe={[start,f(start)]}
1. Choose C = argmin(f-cost) in fringe
2. Add or update C's children to fringe, with associated f-value, remove C from fringe
3. Add C to explored
4. Repeat 1. until C == goal or fringe empty
We will talk more about what heuristics are good or should be used later.

Priority queues can be used to efficiently store and insert states and their f-values into the fringe.
A*

Step: Fringe (argmin)

0: [Arad, 366]

1: [Zerind, 75+374], [Sibiu, 140+253], [Timisoara, 118+329]
2: [Fagaras, 140+99+178], [Rimmicu Vilcea, 140+80+193], [Zerind, 449], [Timisoara, 447], [Oradea, 140+151+380]
3: [Craiova, 140+80+146+160], [Pitesti, 140+80+97+98], [Fagaras, 417], [Zerind, 449], [Timisoara, 447], [Oradea, 671]
4: ... on next slide
A*

4: [Craiova from Rimmicu Vilcea, 526], [Fagaras, 417], [Zerind, 449], [Timisoara, 447], [Oradea, 671], [Craiova from Pitesti, 140+80+97+138+160], [Bucharest from Pitesti, 140+80+97+101+0]

4: [Craiova from Rimmicu Vilcea, 526], [Fagaras, 417], [Zerind, 449], [Timisoara, 447], [Oradea, 671], [Craiova from Pitesti, 615], [Bucharest from Pitesti, 418]

5: [Craiova from Rimmicu Vilcea, 526], [Zerind, 449], [Timisoara, 447], [Oradea, 671], [Craiova from Pitesti, 615], [Bucharest from Pitesti, 418], [Bucharest from Fagaras, 140+99+211+0 = 450]

Goal!
You can choose multiple heuristics (more later) but good ones skew the search to the goal.

You can think circles based on $f$-cost:
- if $h(node) = 0$, $f$-cost are circles
- if $h(node) = \text{very good}$, $f$-cost long and thin ellipse

This can also be though of as topographical maps (in a sense)
\[ h(\text{node}) = 0 \]
(bad heuristic, no goal guidance)

\[ h(\text{node}) = \text{straight line distance} \]
(good heuristic)
A*

Good heuristics can remove “bad” sections of the search space that will not be on any optimal solution (called pruning)

A* is optimal and in fact, no optimal algorithm could expand less nodes (optimally efficient)

However, the time and memory cost is still exponential (memory tighter constraint)
You do it! Find path S -> G

Arrows show children (easier for you)

(see: https://www.youtube.com/watch?v=sAoBeujec74)
Iterative deepening A*  

You can combine iterative deepening with A*  

Idea:  
1. Run DFS in IDS, but instead of using depth as cutoff, use f-cost  
2. If search fails to find goal, increase f-cost to next smallest seen value (above old cost)  

Pros: Efficient on memory  
Cons: Large (LARGE) amount of re-searching
Iterative deepening A*

Consider the following tree and heuristic

Let’s run IDA* on this

<table>
<thead>
<tr>
<th>State</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>7</td>
</tr>
<tr>
<td>A</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
</tr>
</tbody>
</table>
Iterative deepening A*

Iterative deepening, round 1:
Limit = $h(s) = 7$

Run DFS expanding nodes less (or =) limit

Fringe:
1: $(S, 7)$
2: $(A, 10), (B, 9)$
3: $(A, 10)$

This is DFS FILO not finding minimum
Iterative deepening A*

Smallest f-cost above limit in previous search = 9

New limit = 9

1: (S, 7)

2: (A, 10), (B, 9)

3: (A, 10), (C, 14)

4: (A, 10)
Iterative deepening A*

Smallest f-cost above limit in previous search = 10 = limit

1: (S,7)
2: (A,10), (B,9)
3: (A,10), (C,14)
4: (A,10)
5: (B,7), (C,13), (G,16)
6: (B,7), (C,13)
7: (B,7)
8: (C,11)

<table>
<thead>
<tr>
<th>State</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>7</td>
</tr>
<tr>
<td>A</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
</tr>
</tbody>
</table>
Iterative deepening A*

Smallest f-cost above limit in previous search = 11 = limit

... and repeat this process until goal is found

Since search is DFS, memory efficient
One fairly straight-forward modification to A* is simplified memory-bounded A* (SMA*)

Idea:
1. Run A* normally until out of memory
2. Let C = argmax(\(f\)-cost) in the leaves
3. Remove C but store its value in the parent (for re-searching)
4. Goto 1
Here assume you can only hold at most 3 nodes in memory.
SMA*

SMA* is nice as it (like A*) find the optimal solution while keeping re-searching low (given your memory size)

IDA* only keeps a single number in memory, and thus re-searches many times (inefficient use of memory)

Typically there is some time to memory trade-off