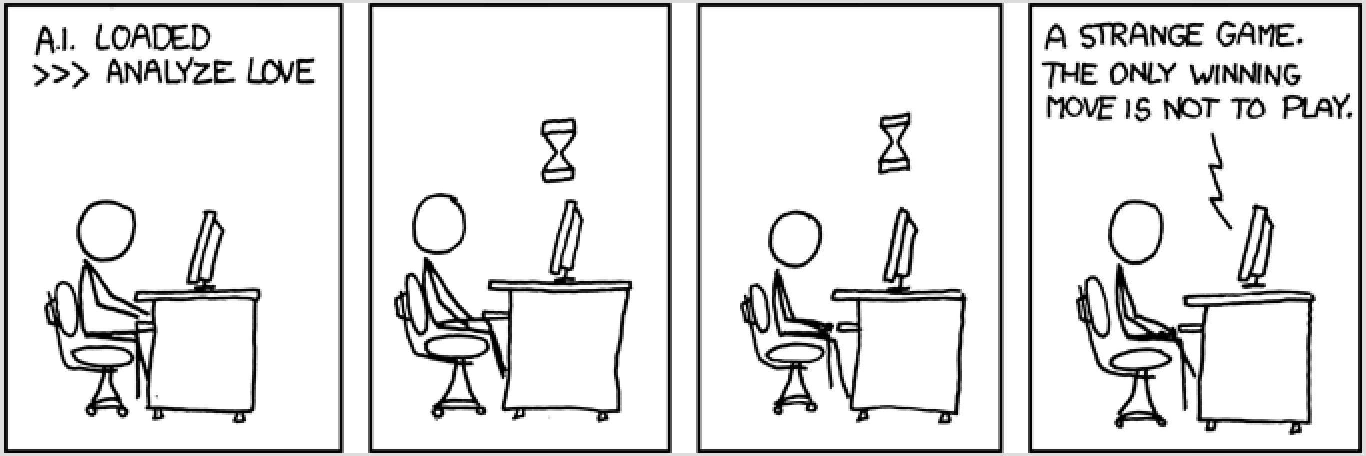
Game theory (Ch. 17.5)

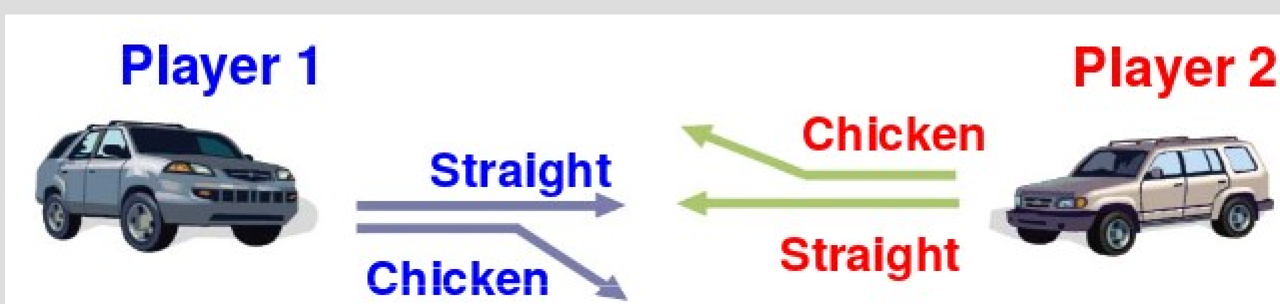


Chicken

What is Nash (pure and mixed) for this game?
What is Pareto optimum?

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Game of Chicken



Chicken

To find Nash, assume we (blue) play S probability p, C prob 1-p

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Column 1 (red=S): p*(-10) + (1-p)*(1)
Column 2 (red=C): p*(-1) + (1-p)*(0)

Intersection: -11*p + 1 = -p, p = 1/10

Conclusion: should always go straight 1/10 and chicken 9/10 the time

Chicken

We can see that 10% straight makes the opponent not care what strategy they use:

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

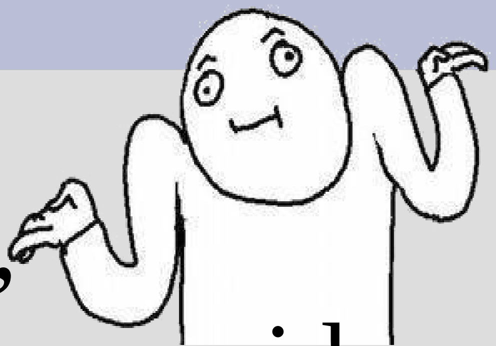
(Red numbers)

100% straight: (1/10)*(-10) + (9/10)*(1) = -0.1

100% chicken: (1/10)*(-1) + (9/10)*(0) = -0.1

50% straight: (0.5)*[(1/10)*(-10) + (9/10)*(1)]
+ (0.5)*[(1/10)*(-1) + (9/10)*(0)]
=(0.5)*[-0.1] + (0.5)*[-0.1] = -0.1

Chicken



	s	c
s	-10, -10	1, -1
c	-1, 1	0, 0

The opponent does
not care about action,
but you still do (never considered our values)

Your rewards, opponent 100% straight:

$(0.1)*(-10) + (0.9)*(-1) = -1.9$

Your rewards, opponent 100% curve:

$(0.1)*(1) + (0.9)*(0) = 0.1$

The opponent also needs to play at your value intersection to achieve Nash

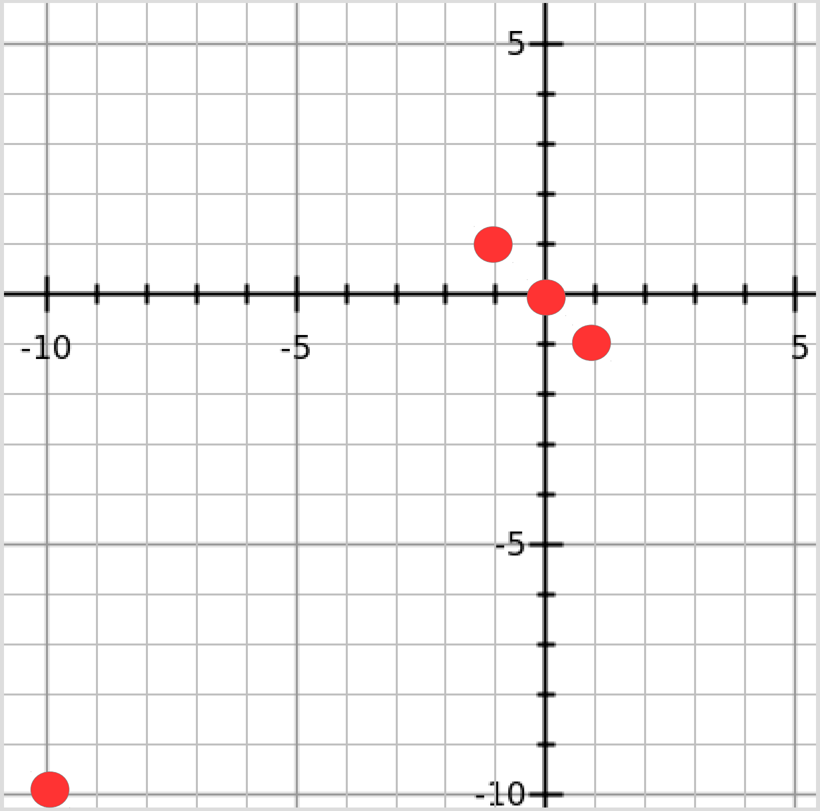
Chicken

Pareto optimum?
All points except (-10,10)

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Going off the definition, P1 loses point if move off (1,-1)
... similar P2 on (-1,1)

At (0,0) there is no point with both vals positive

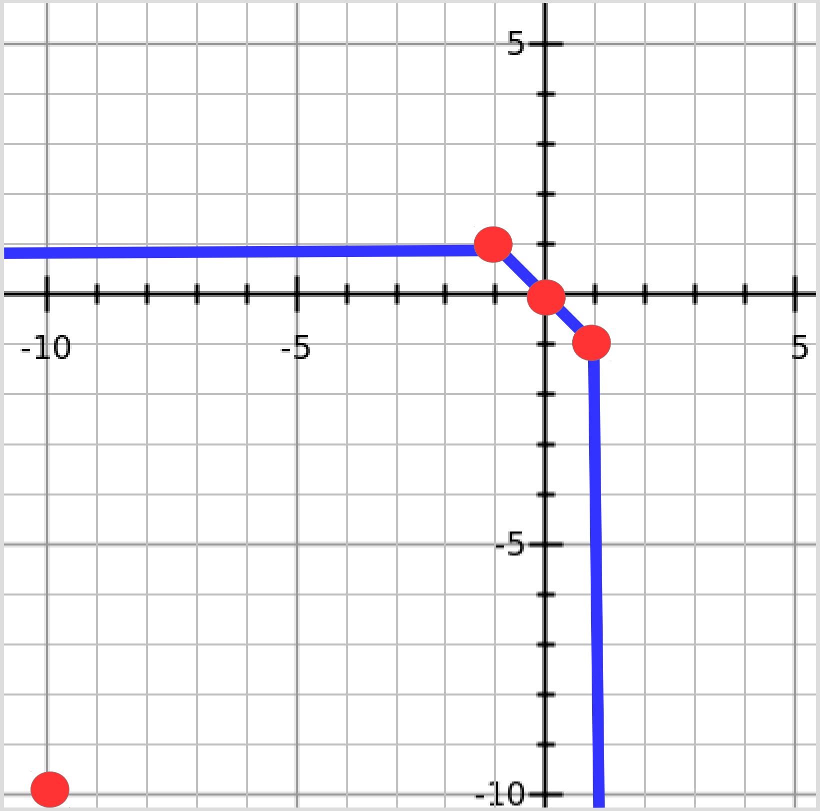


Chicken

We can define a mixed strategy Pareto optimal points

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Can think about this as taking a string from the top right and bringing the it down & left



Stop when string going straight left and down

Pareto on Mixed Strategy

With mixed strategies, this “string” method might eliminate Pareto for pure strategies

Consider the following payoff matrix:

(0, 3)	(1,1)
(3, 0)	(-4, 4)

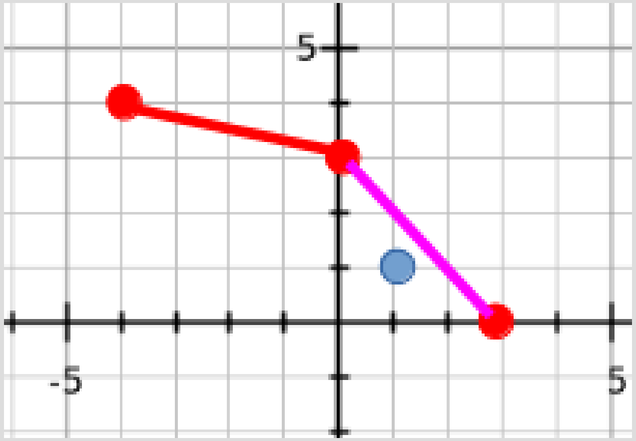
When considering only pure strategies, all four points are Pareto optimal

However, for mixed strategies this is no longer the case

Pareto on Mixed Strategy

If the column player always plays on the left column, this corresponds to the pink line in the graph (depending on the percent row player chooses for top/bottom)

(0, 3)	(1,1)
(3, 0)	(-4, 4)



For example, if the row player does 50% top and 50% bottom, the average payoff will be (1.5, 1.5), which is better for both players than the (1,1) (no longer Pareto)

Find best strategy

We have two actions, so one parameter (p) and thus we look for the intersections of lines

If we had 3 actions (rock-paper-scissors), we would have 2 parameters and look for the intersection of 3 planes (2D)

This can generalize to any number of actions (but not a lot of fun)

		Player 2		
		Stone	Paper	Scissors
Player 1	Stone	$(0, 0)$	$(-1, 1)$	$(1, -1)$
	Paper	$(1, -1)$	$(0, 0)$	$(-1, 1)$
	Scissors	$(-1, 1)$	$(1, -1)$	$(0, 0)$

Find best strategy

Setting up a system of equations is not too bad, but things can become difficult with dominant strategies....

Consider this payoff matrix:

(1,4)	(3,2)	(5,99)
(2,99)	(2,5)	(3,9)
(6,1)	(7,99)	(2,4)

The middle row is dominated by a mixed strategy of the top and bottom (50% top)

Find best strategy

Just like the 2x2 case, you should eliminate the dominated strategy from consideration... which turns this to a 2x3:

(1,4)	(3,2)	(5,99)
~~(2,99)~~	~~(2,5)~~	~~(3,9)~~
(6,1)	(7,99)	(2,4)

However, in the resultant 2x3 there is again a dominated strategy... the left column is dominated by the right column, so you reduce again and end up with:

(3,2)	(5,99)
(7,99)	(2,4)

... which we can solve normally

Find best strategy

The case before this had an obvious dominant strategy, but things can be not so apparent

Consider this payoff matrix:

(13, 1)	(4,11)	(5,10)
(12, 14)	(17,15)	(16,7)
(9,10)	(18,1)	(8,2)

There are no dominant rows/columns (though close), yet the mixed strategy probability is still bogus...

Find best strategy

(13, 1)	(4,11)	(5,10)
(12, 14)	(17,15)	(16,7)
(9,10)	(18,1)	(8,2)

Unfortunately, you have to try all nine 2x2 sub-payoff matrices (much like pure Nash)

However, it is not sufficient to just find a Nash in the sub-2x2, you need to confirm in the 3x3 that the ignored option is not better (else we don't have our "stable" criteria for Nash)

Find best strategy

For example, this 2x2has a Nash with probabilities shown

~~(13, 1)~~	~~(4,11)~~	~~(5,10)~~	
~~(12, 14)~~	(17,15)	(16,7)	1/9
~~(9,10)~~	(18,1)	(8,2)	8/9
	8/9	1/9	

However, if you calculate the value of the column player on the excluded col, it is higher

Center column value: 15*1/9 + 1*8/9 = 2.555
Left column value: 14*1/9 + 10*8/9 = 10.444
... so the column player would want to change

Find best strategy

But at least one Nash has to exist somewhere (and there are no pure strategy Nash)

After checking all possibilities, there turns out to be only one mixed Nash:

	~~(13, 1)~~	~~(4,11)~~	~~(5,10)~~
90%	(12, 14)	(17,15)	~~(16,7)~~
10%	(9,10)	(18,1)	~~(8,2)~~
	25%	75%	

Repeated games

In repeated games, things are complicated

For example, in the basic PD, there is no benefit to “lying”

		PRISONER 2	
		Confess	Lie
PRISONER 1	Confess	-8 , -8	0 , -10
	Lie	-10 , 0	-1 , -1

However, if you play this game multiple times, it would be beneficial to try and cooperate and stay in the [lie, lie] strategy

Repeated games

One way to do this is the <u>tit-for-tat</u> strategy:

1. Play a cooperative move first turn
2. Play the type of move the opponent last played every turn after (i.e. answer competitive moves with a competitive one)

This ensure that no strategy can "take advantage" of this and it is able to reach cooperative outcomes

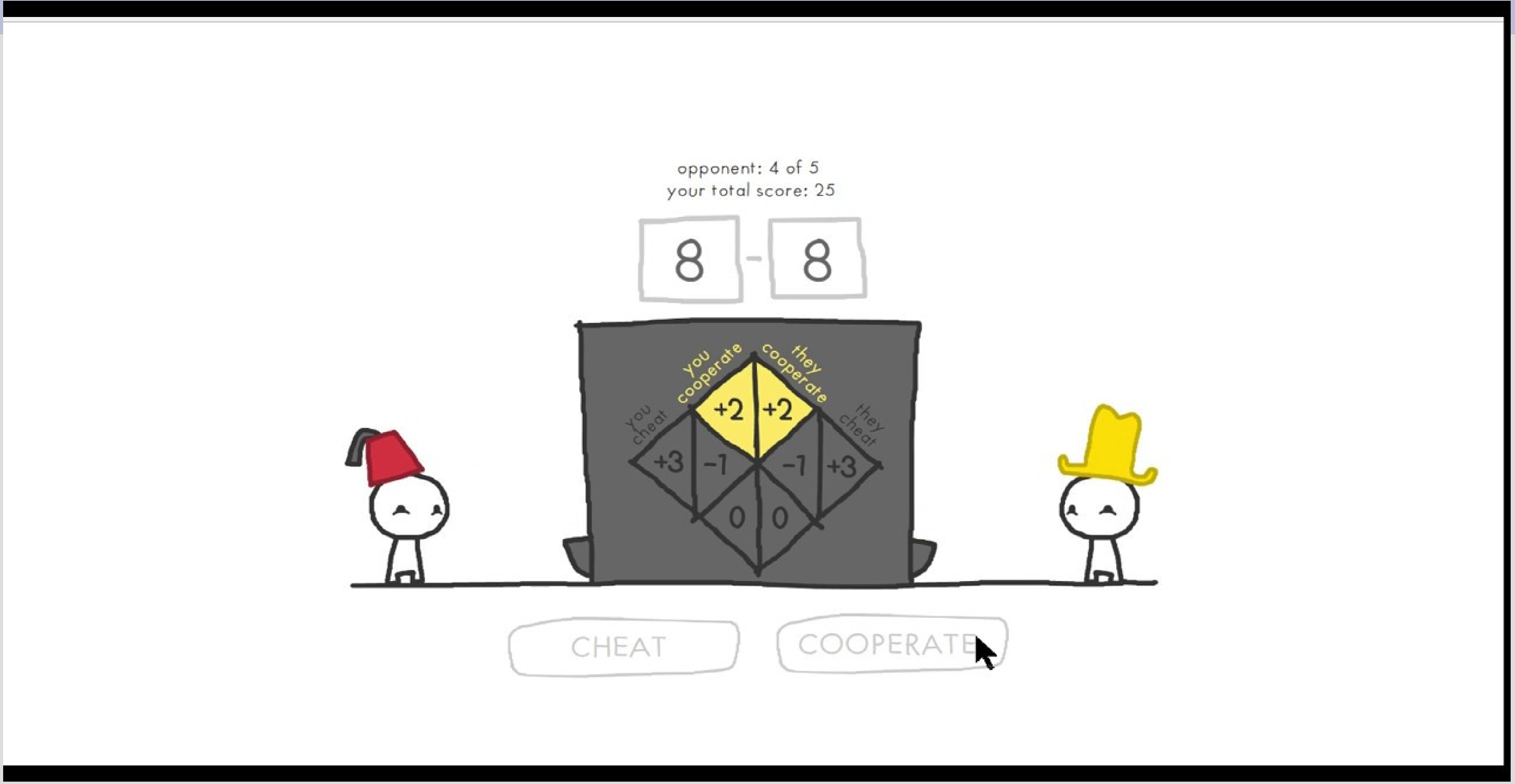
Repeated games

Two “hard” topics (if you are interested) are:

1. We have been talking about how to find best responses, but it is very hard to take advantage if an opponent is playing a sub-optimal strategy

2. How to “learn” or “convince” the opponent to play cooperatively if there is an option that benefits both (yet dominated)

Repeated games



http://ncase.me/trust/