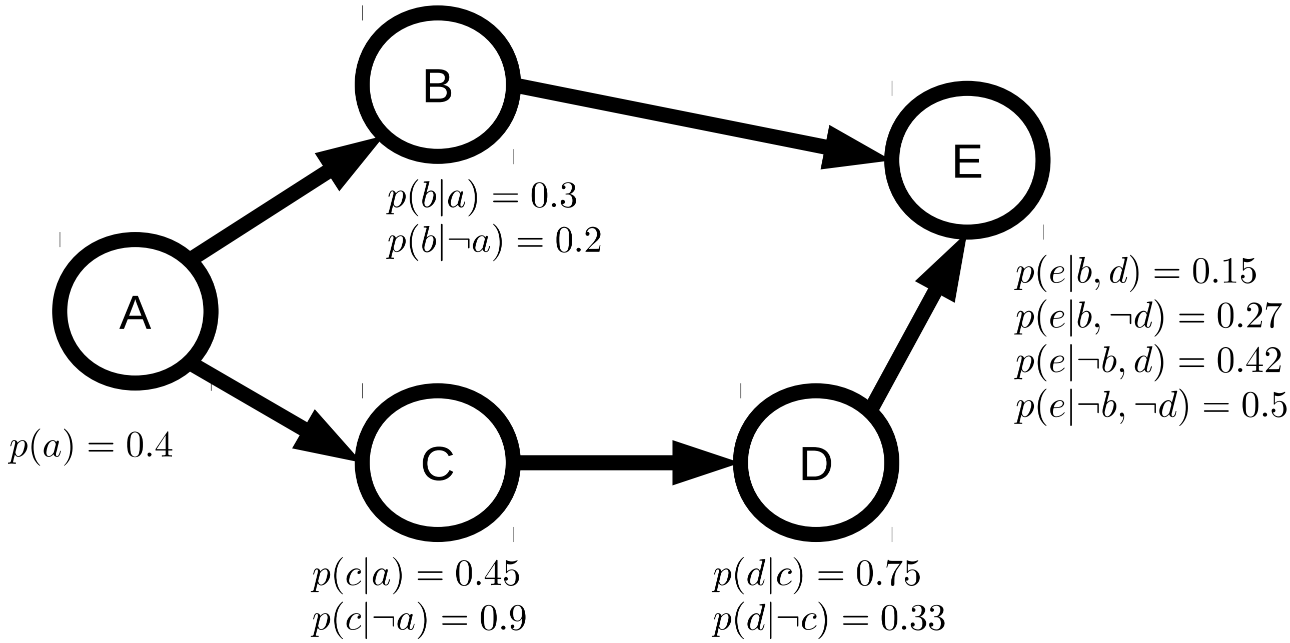
CSci5512, Fall-2021
ASSIGNMENT 1 :
Assigned: 09/30/21 Due: 10/14/21 at 11:55 PM (submit via Canvas, you may scan or take a picture of your paper answers) Please organize your work before submitting.
On all problems you must show work to receive full credit; all answers done individually

Problem 1 & 2 use the following Bayesian Network:



Problem 1. (points)
Write two different ways to find the probability of p(d|b,¬e). using the chain rule and pushing the sums as far right as possible. You may keep your answers in terms of p(variables) rather than putting in numerical values. Which one of these ways is more efficient?

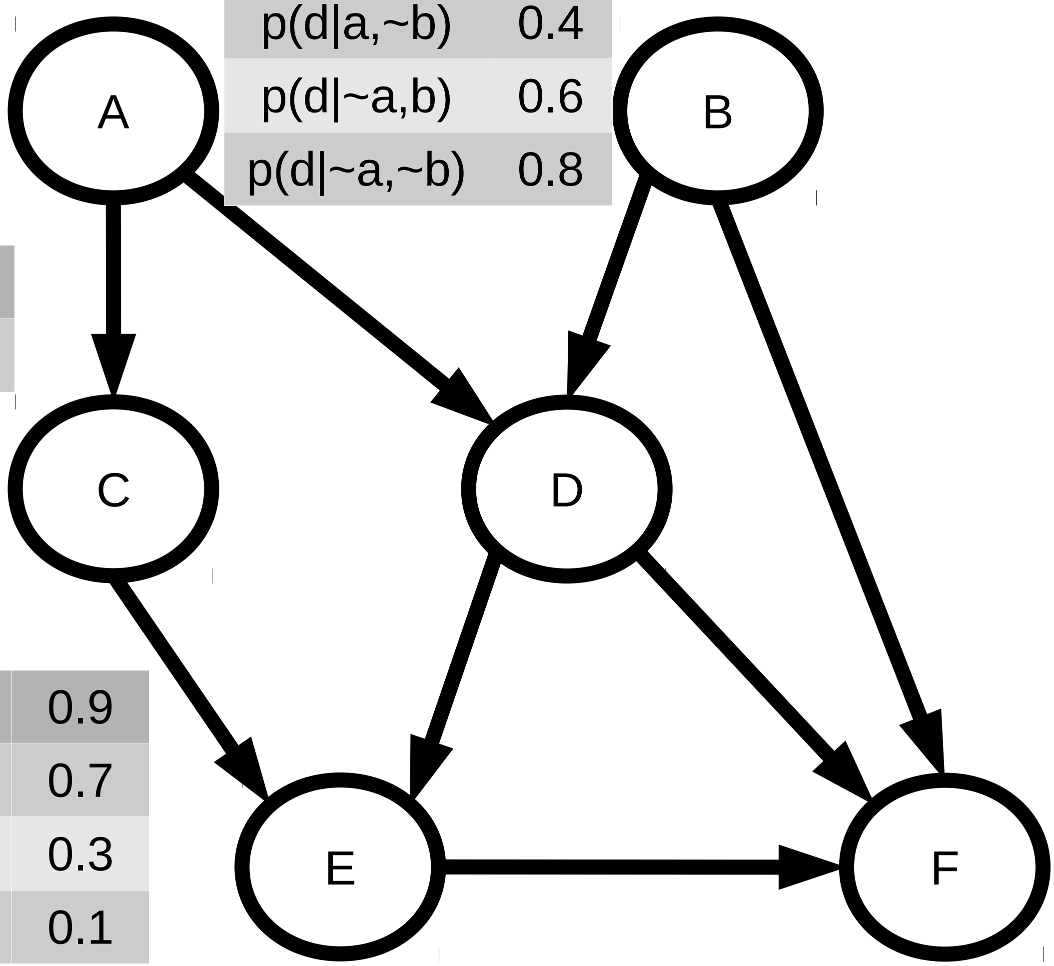
Problem 2. (points)
Use Variable elimination to find p(d|b,¬e) in the above BayesNet (you might want to refer to your answer in problem 1, but it is not necessary).

Problems 3, 4 and 5 refer to this Bayesian Network:

p(a)	0.4

p(d|a,b)	0.2
p(d|a,~b)	0.4
p(d|~a,b)	0.6
p(d|~a,~b)	0.8

p(b)	0.7



p(c|a)	0.9
p(c|~a)	0.3

p(e|c,d)	0.9
p(e|c,~d)	0.7
p(e|~c,d)	0.3
p(e|~c,~d)	0.1

p(f|b,d,e)	0.2
p(f|b,d,~e)	0.1
p(f|b,~d,e)	0.4
p(f|b,~d,~e)	0.3
p(f|~b,d,e)	0.6
p(f|~b,d,~e)	0.7
p(f|~b,~d,e)	0.8
p(f|~b,~d,~e)	0.5

Problem 3. (15 points)
(Part 1) Use rejection sampling to estimate p(d|¬b,e,f). You should write your own code to solve this problem (not find an already implemented version). You may use some code existing code (such as a “Node” class or something), but the main algorithm should be coded on your own. Submit both your final answer to this part, and your source code as a supplementary file. A list of languages we will accept are: C++, Java, Python, OCaml, and Matlab. If you want to use a language that is not on this list, please email me (jparker@cs.umn.edu) and we can see if it is possible.

(Part 2) Approximately how many samples do you need to use in order to find p(d|¬b,e,f) accurate within 2 significant figures/digits after rounding? (This approximation can either be derived mathematically or empirically.)

Problem 4. (15 points)
(Part 1) Again estimate p(d|¬b,e,f), but using likelihood weighting this time. (Same rules as previous problem.)

(Part 2) Approximately how many samples do you need to use in order to find p(d|¬b,e,f) accurate within 2 significant figures/digits after rounding?

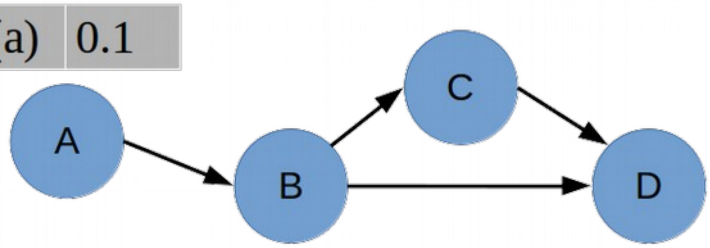
Problem 5. (15 points)
(Part 1) Again estimate p(d|¬b,e,f), but using Gibbs sampling. (Same rules as previous problem.)

(Part 2) Approximately how many samples do you need to use in order to find p(d|¬b,e,f) accurate within 2 significant figures/digits after rounding?

Problem 6. (10 points)
Consider the network below (should be familiar). Assuming we wan to find p(¬a,¬b,¬d|c) using Gibbs sampling. In-class we discussed how you can treat Gibbs sampling as walking around a graph as a Markov chain.

P(c|b)	0.4
P(c|¬b)	0.5

P(a)	0.1



P(b|a)	0.2
P(b|¬a)	0.3

P(d|b,c)	0.25
P(d|b,¬c)	1.0
P(d|¬b,c)	0.15
P(d|¬b,¬c)	0.05

(1) Specifically write out all 8 states and the transition probabilities between them (i.e. edge

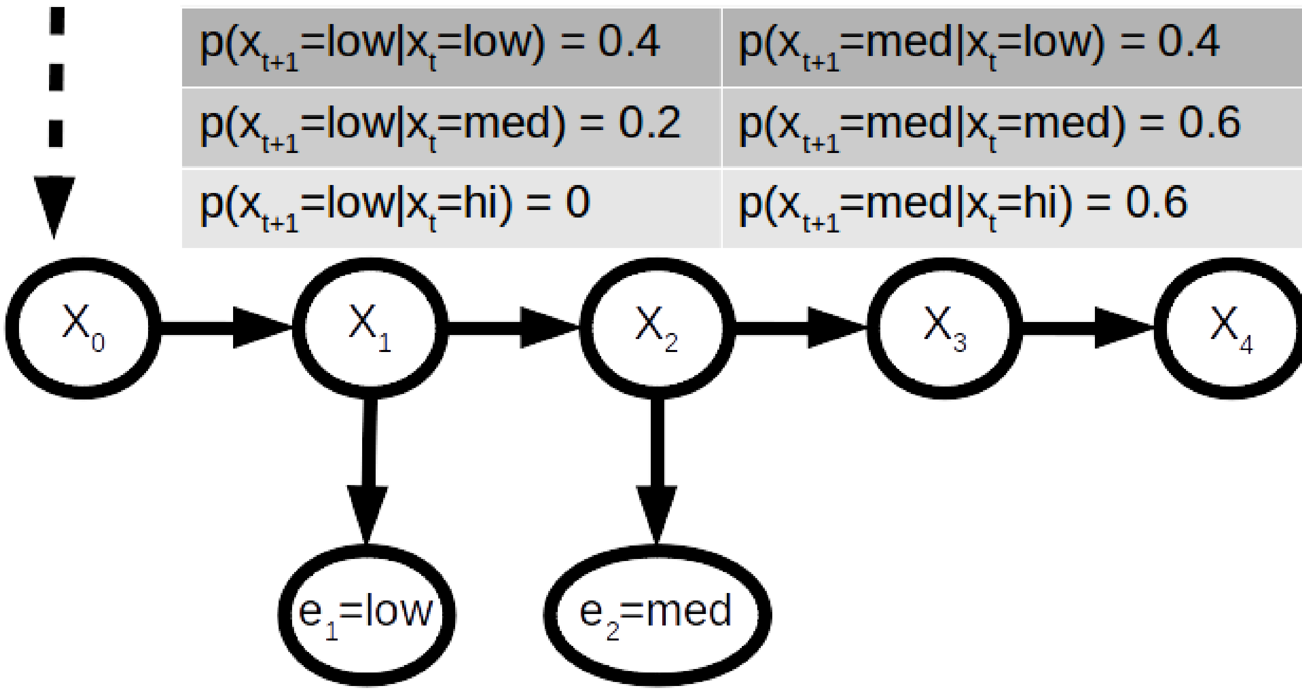
probabilities) for this problem. Assume that each “free” variable (a,b,d) are picked equally at random to have a chance to change (1/3 prob).

(2) Assume p(a,b,d|c) ≈ 0.00424 (approximation), use our stationary distribution property (“flow across both directions is the same”) to solve for p(¬a,¬b,¬d|c). (Do not solve or approximate this probability from scratch. Use the stationary distribution property specific to Gibbs sampling.)

Problems 7 & 8 deal with this HMM: (note: the variables have 3 possible values of low/medium/high rather than our normal T/F)

$p(x_0=low) = 0.3$	$p(x_0=med) = 0.4$

$p(x_{t+1}=low \mid x_t=low) = 0.4$	$p(x_{t+1}=med \mid x_t=low) = 0.4$
$p(x_{t+1}=low \mid x_t=med) = 0.2$	$p(x_{t+1}=med \mid x_t=med) = 0.6$
$p(x_{t+1}=low \mid x_t=hi) = 0$	$p(x_{t+1}=med \mid x_t=hi) = 0.6$



$p(e_t=low \mid x_t=low) = 0.8$	$p(e_t=med \mid x_t=low) = 0.2$
$p(e_t=low \mid x_t=med) = 0.1$	$p(e_t=med \mid x_t=med) = 0.8$
$p(e_t=low \mid x_t=hi) = 0.1$	$p(e_t=med \mid x_t=hi) = 0.4$

Problem 7. (15 points)
Use filtering to estimate p(x2=high | e1 = low, e2 = medium).

Problem 8. (5 points)
Use prediction to estimate p(x4=low | e1 = low, e2 = medium).