Simple Bayes Net

Scenario: "Suppose when I go home at night, I want to know if the family is at home before trying the doors (perhaps the most convenient door is double-locked when nobody is at home). Now often when my wife leaves the house she turns on the outside light, but sometimes she turns it on when expecting a guest. When nobody is home, the dog is put in the back yard, but the same is true if the dog has bowel problems. When the dog is in back I can hear her barking, but sometimes I get confused with other dogs barking."*

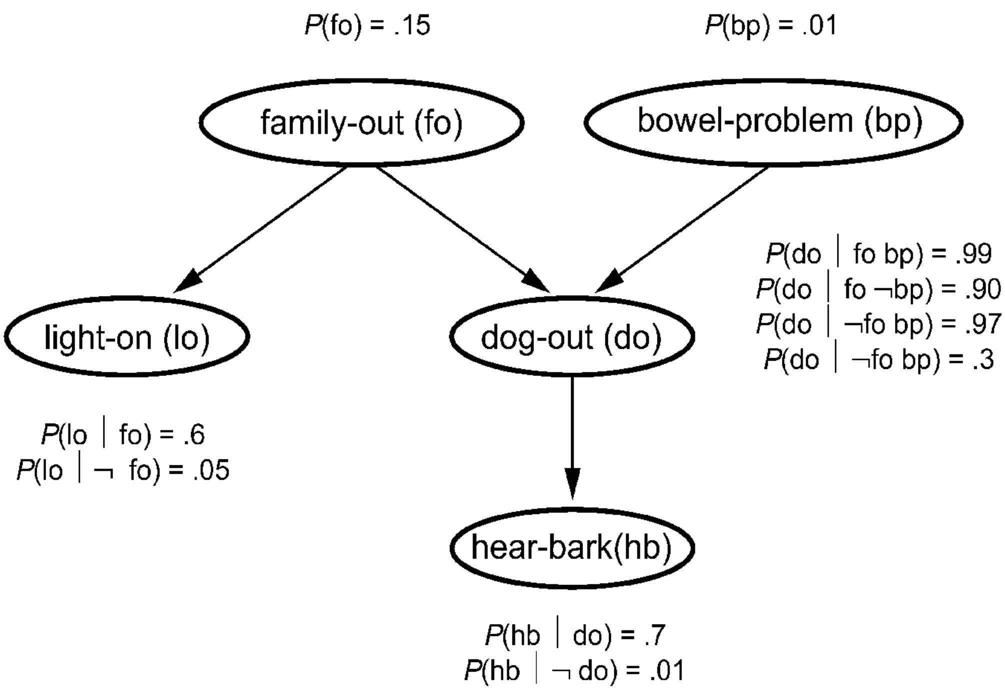

Diagram of Bayes network for barking dog.*

Conditional probabilities along each link:	**Complementary probabilities along each link:**
P(family-out) = .15	P(no family-out) = .85
P(bowel-problem) = .01,	P(no bowel-problem) = .99,
P(dog-out|family-out & bowel-prob.) = .99	P(no dog-out|family-out & bowel-prob.) = .01
P(dog-out|family-out & no bowel-prob.) = .90	P(no dog-out|family-out & no bowel-prob.) = .10
P(dog-out|no family-out & bowel-prob.) = .97	P(no dog-out|no family-out & bowel-prob.) = .03
P(dog-out|no family-out & no bowel-prob.) = .3	P(no dog-out|no family-out & no bowel-prob.) = .7
P(hear-bark|dog-out) = .7	P(no hear-bark|dog-out) = .3
P(hear-bark|no dog-out) = .01	P(no hear-bark|no dog-out) = .99
P(light-on|family-out) = .6	P(no light-on|family-out) = .4
P(light-on|no family-out) = .05	P(no light-on|no family-out) = .95

*extracted by D. Boley from "Bayesian Networks without Tears" by Eugene Charniak, *AI Magazine* 1991.

Table of Conditional Probabilites.

Each probability is conditioned on all the conditions to its left. When independent, the conditional probabilities just repeat.

	family out	prior prob.	bowel problem	prior prob.	dog out	cond. prob.	hear bark	cond. prob.	light on	cond. prob.
0	no	.85	no	.99	no	.70	no	.99	no	.95
1	no	↓	no	↓	no	↓	no	↓	yes	.05
2	no		no		no		yes	.01	no	.95
3	no		no		no		yes	↓	yes	.05
4	no		no		yes	.30	no	.30	no	.95
5	no		no		yes	↓	no	↓	yes	.05
6	no		no		yes		yes	.70	no	.95
7	no		no		yes		yes	↓	yes	.05
8	no		yes	.01	no	.03	no	.99	no	.95
9	no		yes	↓	no	↓	no	↓	yes	.05
10	no		yes		no		yes	.01	no	.95
11	no		yes		no		yes	↓	yes	.05
12	no		yes		yes	.97	no	.30	no	.95
13	no		yes		yes	↓	no	↓	yes	.05
14	no		yes		yes		yes	.70	no	.95
15	no		yes		yes		yes	↓	yes	.05
16	yes	.15	no	.99	no	.01	no	.99	no	.40
17	yes	↓	no	↓	no	↓	no	↓	yes	.60
18	yes		no		no		yes	.01	no	.40
19	yes		no		no		yes	↓	yes	.60
20	yes		no		yes	.99	no	.30	no	.40
21	yes		no		yes	↓	no	↓	yes	.60
22	yes		no		yes		yes	.70	no	.40
23	yes		no		yes		yes	↓	yes	.60
24	yes		yes	.01	no	.01	no	.99	no	.40
25	yes		yes	↓	no	↓	no	↓	yes	.60
26	yes		yes		no		yes	.01	no	.40
27	yes		yes		no		yes	↓	yes	.60
28	yes		yes		yes	.99	no	.30	no	.40
29	yes		yes		yes	↓	no	↓	yes	.60
30	yes		yes		yes		yes	.70	no	.40
31	yes		yes		yes		yes	↓	yes	.01

Table of Joint Probabilites for All Possible Combinations of Events

Each probability is the joint probability of all conditions to its left.

	family out	prior prob.	bowel problem	joint prob.	dog out	joint prob.	hear bark	joint prob.	light on	joint prob.
0	no	.8500	no	.8415	no	.58905	no	.5831595	no	.554001525
1	no	↓	no	↓	no	↓	no	↓	yes	.029157975
2	no		no		no		yes	.0058905	no	.005595975
3	no		no		no		yes	↓	yes	.000294525
4	no		no		yes	.25245	no	.075735	no	.071948250
5	no		no		yes	↓	no	↓	yes	.00378675
6	no		no		yes		yes	.176715	no	.16787925
7	no		no		yes		yes	↓	yes	.00883575
8	no		yes	.0085	no	2.55e-4	no	2.5245e-4	no	2.398275e-4
9	no		yes	↓	no	↓	no	↓	yes	1.26225e-5
10	no		yes		no		yes	2.55e-6	no	2.4225e-6
11	no		yes		no		yes	↓	yes	1.275e-7
12	no		yes		yes	.008245	no	.0024735	no	.002349825
13	no		yes		yes	↓	no	↓	yes	1.23675e-4
14	no		yes		yes		yes	.0057715	no	.005482925
15	no		yes		yes		yes	↓	yes	2.88575e-4
16	yes	.1500	no	.1485	no	.01485	no	.0147015	no	0.0058806
17	yes	↓	no	↓	no	↓	no	↓	yes	.0088209
18	yes		no		no		yes	1.485e-4	no	5.94e-5
19	yes		no		no		yes	↓	yes	8.91e-5
20	yes		no		yes	.13365	no	.040095	no	0.016038
21	yes		no		yes	↓	no	↓	yes	.024057
22	yes		no		yes		yes	.093555	no	.037422
23	yes		no		yes		yes	↓	yes	.056133
24	yes		yes	.0015	no	1.5e-5	no	1.485e-5	no	5.94e-6
25	yes		yes	↓	no	↓	no	↓	yes	8.91e-6
26	yes		yes		no		yes	1.5e-7	no	6.0e-8
27	yes		yes		no		yes	↓	yes	9.0e-8
28	yes		yes		yes	.001485	no	4.455e-4	no	1.782e-4
29	yes		yes		yes	↓	no	↓	yes	2.673e-4
30	yes		yes		yes		yes	.0010395	no	4.158e-4
31	yes		yes		yes		yes	↓	yes	6.237e-4

To figure out what is the probability that the dog is out given that the family is out, the light is off and you do not hear a bark....

1. Since we don't know if there is a bowel-problem, we use the given prior for this attribute.

2. The starting information restricts us to the states 16, 20, 24, 28. Of these four states, only two, 20 and 28, correspond to the dog being out. So the probability of being in state 20 or 28 given that we are in one of the states 16, 20, 24, 28 is

$$\frac{0.016038 + 1.782\text{e-}4}{0.0058806 + 0.016038 + 5.94\text{e-}6 + 1.782\text{e-}4} = \frac{.0162162}{.02210274} = .73367374361730717549.$$

3. We can also figure it out using Bayes formula repeatedly. From the parents of "dog-out" we have that

$$\begin{aligned} P(\text{dog-out}|\text{family-out}) &= P(\text{dog-out}|\text{family-out}\,\&\,\text{bowel-problem}) \cdot P(\text{bowel-problem}) \\ &\quad + P(\text{dog-out}|\text{family-out}\,\&\,\text{no bowel-problem}) \cdot P(\text{no bowel-problem}) \\ &= .99 \cdot .01 + .90 \cdot .99 = .9009. \end{aligned}$$

This can be considered as a "local prior" for the event "dot-out" which we will denote as $\widehat{P}(\text{dog-out})$.

4. The "light-out" event is conditionally independent (conditioned on "family-out") of the "dog-out" event, so we can ignore it here. Using Bayes theorem for the child of "dog-out" yields:

$$\begin{aligned} & P(\text{dog-out}|\text{no hear-bark}) \\ &= \frac{P(\text{no hear-bark}|\text{dog-out}) \cdot \widehat{P}(\text{dog-out})}{P(\text{no hear-bark})} \\ &= \frac{P(\text{no hear-bark}|\text{dog-out}) \cdot \widehat{P}(\text{dog-out})}{P(\text{no hear-bark}\,\&\,\text{dog-out}) + P(\text{no hear-bark}\,\&\,\text{no dog-out})} \\ &= \frac{P(\text{no hear-bark}|\text{dog-out}) \cdot \widehat{P}(\text{dog-out})}{P(\text{no hear-bark}|\text{dog-out}) \cdot \widehat{P}(\text{dog-out}) + P(\text{no hear-bark}|\text{no dog-out}) \cdot \widehat{P}(\text{no dog-out})} \\ &= \frac{(1-.7) \cdot .9009}{(1-.7) \cdot .9009 + (1-.01) \cdot (1-.9009)} = .733673743617307175. \end{aligned}$$