CSci5512, Spring-2019
ASSIGNMENT 1 :
Assigned: 02/05/19 Due: 02/17/19 at 11:55 PM (submit via Canvas, you may scan or take a picture of your paper answers) Submit only pdf or txt files
On all problems you must show work to receive full credit; all answers found individually

Problem 1. (15 points)
In-class we discussed how instead of using Bayes rule to solve P(a|b) (i.e. P(a|b) = P(b|a)P(a)/P(b)) that instead you could ignore the denominator and instead find P(¬a|b) and normalize. Prove that these methods are theoretically equivalent (i.e. formally prove that this trick will always work).

Problem 2. (15 points)
In-class we did an example where I said P(a) = 0.2, P(b) = 0.3, P(a or b) = 0.1. I claimed these probabilities were not consistent with each other. What property is violated? Prove this property using only these five facts that I gave in-class:

(1) $0 \leq P(\omega) \leq 1$

(2) $\sum_{\omega \in \Omega} P(\omega) = 1$, where Ω is the set of all possible outcomes

(3) $P(a) + P(\neg a) = 1$

(4) $P(a \text{ or } b) = P(a) + P(b) - P(a,b)$

(5) $P(a) = \sum_{b} P(a,b)$

Problem 3, 4 & 5 use this table:

Tables for P(a,b,c)

P(a,b,c)	a	¬a
b	0.018837	0.126324
¬b	0.063063	0.160776

when c

P(a,b,¬c)	a	¬a
b	0.011063	0.256476
¬b	0.037037	0.326424

when ¬c

same

$P(a, b, c) = 0.018837$

$P(a, b, \neg c) = 0.011063$

$P(a, \neg b, c) = 0.063063$

$P(a, \neg b, \neg c) = 0.037037$

$P(\neg a, b, c) = 0.126324$

$P(\neg a, b, \neg c) = 0.256476$

$P(\neg a, \neg b, c) = 0.160776$

$P(\neg a, \neg b, \neg c) = 0.326424$

Problem 3. (20 points)
Find the following probabilities using the table above.
(1) P(a,b)
(2) P(a,b | c)
(3) P(c | ¬a)
(4) P(b)

Problem 4. (20 points)
Using the same table as problem 3, are any of the variables independent? Are any of the variables conditionally independent? (Show a the rationale for your statements.)

Problem 5. (20 points)
Using the same table as problems 3 and 4, build a Bayesian network (graph and tables) accurately representing the variables in the table.

(1) Give the most **efficiently** Bayesian network (least amount of probabilities)
(2) Give the most **inefficient** Bayesian network (maximum amount of probabilities to define network without giving the probabilities for opposite events (e.g. can't give both P(a|b) and P(¬a|b))

Problem 6. (20 points)
Pretend there is a slot-machine at the Casino that works as following: 10% of the time it gives a jackpot of $100, 30% of the time it gives a medium reward of $30, 50% of the time it gives a low reward of $5 and 10% of the time you get nothing.

(1) Represent the slot machine as a random variable.
(2) What price should the Casino attach to play this machine?
(3) What is the probability that you get at least one reward from playing the slot-machine 5 times?

Problem 7. (5 points)
Suppose a nasty employee modifies the slot-machine from the previous problem. If a jackpot is gotten, the next pull of the slot machine will result in 50% of the time giving the low reward ($5) and 50% of the time giving nothing. What expected amount of money out of two plays of this new slot machine (assuming no jackpot was gotten before the start of these two plays)?