

ASSIGNMENT 1 :

Assigned: 02/05/19 Due: 02/17/19 at 11:55 PM (submit via Canvas, you may scan or take a picture of your paper answers) Submit only pdf or txt files

On all problems you must show work to receive full credit; all answers found individually

Problem 1. (15 points)

In-class we discussed how instead of using Bayes rule to solve $P(a|b)$ (i.e. $P(a|b) = P(b|a)P(a)/P(b)$) that instead you could ignore the denominator and instead find $P(\neg a|b)$ and normalize. Prove that these methods are theoretically equivalent (i.e. formally prove that this trick will always work).

Problem 2. (15 points)

In-class we did an example where I said $P(a) = 0.2$, $P(b) = 0.3$, $P(a \text{ or } b) = 0.1$. I claimed these probabilities were not consistent with each other. What property is violated? Prove this property using only these five facts that I gave in-class:

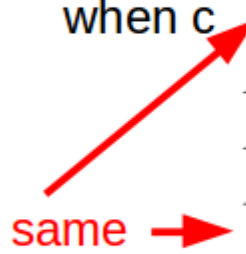
- (1) $0 \leq P(\omega) \leq 1$
- (2) $\sum_{\omega \in \Omega} P(\omega) = 1$, where Ω is the set of all possible outcomes
- (3) $P(a) + P(\neg a) = 1$
- (4) $P(a \text{ or } b) = P(a) + P(b) - P(a, b)$
- (5) $P(a) = \sum_b P(a, b)$

Problem 3, 4 & 5 use this table:

Tables for $P(a,b,c)$

$P(a,b,c)$	a	$\neg a$	$P(a,b,\neg c)$	a	$\neg a$
b	0.018837	0.126324	b	0.011063	0.256476
$\neg b$	0.063063	0.160776	$\neg b$	0.037037	0.326424

when c when $\neg c$



$P(a, b, c) = 0.018837$
 $P(a, b, \neg c) = 0.011063$
 $P(a, \neg b, c) = 0.063063$
 $P(a, \neg b, \neg c) = 0.037037$
 $P(\neg a, b, c) = 0.126324$
 $P(\neg a, b, \neg c) = 0.256476$
 $P(\neg a, \neg b, c) = 0.160776$
 $P(\neg a, \neg b, \neg c) = 0.326424$

Problem 3. (20 points)

Find the following probabilities using the table above.

- (1) $P(a,b)$
- (2) $P(a,b | c)$
- (3) $P(c | \neg a)$
- (4) $P(b)$

Problem 4. (20 points)

Using the same table as problem 3, are any of the variables independent? Are any of the variables conditionally independent? (Show a the rationale for your statements.)

Problem 5. (20 points)

Using the same table as problems 3 and 4, build a Bayesian network (graph and tables) accurately representing the variables in the table.

- (1) Give the most **efficiently** Bayesian network (least amount of probabilities)
- (2) Give the most **inefficient** Bayesian network (maximum amount of probabilities to define network without giving the probabilities for opposite events (e.g. can't give both $P(a|b)$ and $P(\neg a|b)$))

Problem 6. (20 points)

Pretend there is a slot-machine at the Casino that works as following: 10% of the time it gives a jackpot of \$100, 30% of the time it gives a medium reward of \$30, 50% of the time it gives a low reward of \$5 and 10% of the time you get nothing.

- (1) Represent the slot machine as a random variable.
- (2) What price should the Casino attach to play this machine?
- (3) What is the probability that you get at least one reward from playing the slot-machine 5 times?

Problem 7. (5 points)

Suppose a nasty employee modifies the slot-machine from the previous problem. If a jackpot is gotten, the next pull of the slot machine will result in 50% of the time giving the low reward (\$5) and 50% of the time giving nothing. What expected amount of money out of two plays of this new slot machine (assuming no jackpot was gotten before the start of these two plays)?