## Planning (Ch. 10)



## Graph Plan

## Consider this problem:

Initial: Sleepy $(m e) \wedge$ Hungry (me)
Goal: $\neg \operatorname{Sleepy}(m e) \wedge \neg$ Hungry $(m e)$
Action Eat $(x)$, Action( Coffee (x),
Precondition: Hungry $(x)$, Precondition: ,
Effect: $\neg H u n g r y(x))$
Effect: $\neg \operatorname{Sleepy}(x))$
Action(Sleep (x),
Precondition: Sleepy $(x) \wedge \neg H u n g r(x)$,
Effect: $\neg \operatorname{Sleepy}(x) \wedge H u n g r y(x))$

## Mutexes: actions

Mutex Action rules:

1. $x \in E f f e c t(A 1) \wedge \neg x \in E f f e c t(A 2)$
2. $x \in \operatorname{Pre}(A 1) \wedge \neg x \in E f f e c t(A 2)$
3. $x \in \operatorname{Pre}(A 1) \wedge \neg x \in \operatorname{Pre}(A 2)$


## Mutexes: states

There are 2 rules for states, but unlike action-mutexes they can change across levels

1. Opposite relations are mutexes ( x and $\neg \mathrm{x}$ )
2. If there are mutexes between all possible actions that "lead" to a pair of states...

Two ways that "leading" can be in mutex: 2.1. Actions are in mutex
2.2. Preconditions of action pair are in mutex

## Mutexes: states

Another way to compute state mutexes:
(1) Add mutexes between all pairs in state (2) If any pair of actions can lead to this pair of relationships, un-mutex them

Recap:
If any valid pair of actions = no mutex
All ways of reaching invalid = mutex

## Mutexes: states

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This mutex will be gone on the next level (as you can
Sl has mutex with both E and $\mathrm{NoOp}(\neg \mathrm{H})$

## Mutexes: states

## 1. Opposite relations are mutexes ( $x$ and $\urcorner x$ )

 2. If there are mutexes between all possible actions that lead to a pair of states

## Mutexes: actions

## Consider...

Initial: $\neg$ Money $\wedge \neg \operatorname{Smart} \wedge \neg D e b t$ Goal: $\neg$ Money $\wedge$ Smart $\wedge \neg D e b t$

Action (School,

Effect: Debt $\wedge$ Smart) Effect: Money $\wedge \neg$ Smart)

Precondition: Money,
Effect: $\neg$ Money $\wedge \neg D e b t)$

## Mutexes: actions <br>  <br> ${ }_{7} \mathrm{M} \longrightarrow \mathrm{M} \longrightarrow \mathrm{M}$





## GraphPlan

GraphPlan can be computed in $\mathrm{O}\left(\mathrm{n}(\mathrm{a}+\mathrm{l})^{2}\right)$, where $n=$ levels before convergence $\mathrm{a}=$ number of actions l = number of relations/literals/states (square is due to needing to check all pairs)

The original planning problem is PSPACE, which is known to be harder than NP

## GraphPlan: states

## Let's consider this problem: <br> Initial: Clean $\wedge$ Garbage $\wedge$ Quiet

Goal: Food $\wedge \neg$ Garbage $\wedge$ Present

## Action: ( MakeFood, <br> Action: ( Takeout,

Precondition: Clean, Effects: Food)

## Action: (Wrap,

 Precondition: Quiet, Effects: Present)Precondition: Garbage,
Effects: $\neg$ Garbage $\wedge \neg$ Clean $)$

## Action: ( Dolly,

Precondition: Garbage,
Effects: $\neg$ Garbage $\wedge \neg$ Quiet)

## GraphPlan: states



# Take out one more level 

## Mutexes



Possible state pairs:

F, C C, Q

$$
F, 7 C \quad C, \not \subset Q
$$

F, G C, P
$\mathrm{F}, 7 \mathrm{G} 7 \mathrm{C}, \mathrm{G}$
F, Q
F,$\rceil \mathrm{Q} \not \neg \mathrm{C}, \mathrm{Q}$
F, P
$\rightarrow \mathrm{C}, \mathrm{p} \mathrm{Q}$
C, C
${ }_{7} \mathrm{C}, \mathrm{P}$
C, G ... (more)

## Mutexes




## GraphPlan as heuristic

GraphPlan is optimistic, so if any pair of goal states are in mutex, the goal is impossible

3 basic ways to use GraphPlan as heuristic: (1) Maximum level of all goals
(2) Sum of level of all goals (not admissible)
(3) Level where no pair of goals is in mutex
(1) and (2) do not require any mutexes, but are less accurate (quick 'n' dirty)

## GraphPlan as heuristic

For heuristics (1) and (2), we relax as such: 1. Multiple actions per step, so can only take fewer steps to reach same result
2. Never remove any states, so the number of possible states only increases

This is a valid simplification of the problem, but it is often too simplistic directly

## GraphPlan as heuristic

Heuristic (1) directly uses this relaxation and finds the first time when all 3 goals appear at a state level
(2) tries to sum the levels of each individual first appearance, which is not admissible (but works well if they are independent parts)

Our problem: goal=\{Food, $\rceil$ Garbage, Present $\}$ First appearance: $F=1,\rceil G=1, P=1$

## GraphPlan: states

Level 0:
Heuristic (1):
$\operatorname{Max}(1,1,1)=1$

Heuristic (2): $1+1+1=3$

## GraphPlan as heuristic

Often the problem is too trivial with just those two simplifications

So we add in mutexes to keep track of invalid pairs of states/actions

This is still a simplification, as only impossible state/action pairs in the original problem are in mutex in the relaxation

## GraphPlan as heuristic

Heuristic (3) looks to find the first time none of the goal pairs are in mutex

For our problem, the goal states are: (Food, $\rceil$ Garbage, Present)

So all pairs that need to have no mutex: (F, $\rceil \mathrm{G}),(\mathrm{F}, \mathrm{P}),( \rceil \mathrm{G}, \mathrm{P})$


## Finding a solution

GraphPlan can also be used to find a solution:
(1) Converting to a Constraint Sat. Problem (2) Backwards search

Both of these ways can be run once GraphPlan has all goal pairs not in mutex (or converges)

Additionally, you might need to extend it out a few more levels further to find a solution (as GraphPlan underestimates)

## GraphPlan as CSP

## Variables $=$ states, Domains $=$ actions to there Constraints = mutexes \& preconditions


(a) Planning Graph

$$
\begin{aligned}
\text { Variables: } & G_{1}, \cdots, G_{4}, P_{1} \cdots P_{6} \\
\text { Domains: } & G_{1}:\left\{A_{1}\right\}, G_{2}:\left\{A_{2}\right\} G_{3}:\left\{A_{3}\right\} G_{4}:\left\{A_{4}\right\} \\
& P_{1}:\left\{A_{5}\right\} P_{2}:\left\{A_{6}, A_{11}\right\} P_{3}:\left\{A_{7}\right\} P_{4}:\left\{A_{8}, A_{9}\right\} \\
& P_{5}:\left\{A_{10}\right\} P_{6}:\left\{A_{10}\right\}
\end{aligned}
$$

Constraints (normal): $P_{1}=A_{5} \Rightarrow P_{4} \neq A_{9}$

$$
\begin{aligned}
& P_{2}=A_{6} \Rightarrow P_{4} \neq A_{8} \\
& P_{2}=A_{11} \Rightarrow P_{3} \neq A_{7}
\end{aligned}
$$

Constraints (Activity): $G_{1}=A_{1} \Rightarrow$ Active $\left\{P_{1}, P_{2}, P_{3}\right\}$

$$
\begin{aligned}
& G_{2}=A_{2} \Rightarrow \text { Active }\left\{P_{4}\right\} \\
& G_{3}=A_{3} \Rightarrow \text { Active }\left\{P_{5}\right\} \\
& G_{4}=A_{4} \Rightarrow \text { Active }\left\{P_{1}, P_{6}\right\}
\end{aligned}
$$

Init State: Active $\left\{G_{1}, G_{2}, G_{3}, G_{4}\right\}$
(b) DCSP
from Do \& Kambhampati

## Finding a solution

For backward search, attempt to find arrows back to the initial state(without conflict/mutex)

Start by finding actions that satisfy all goal conditions, then recursively try to satisfy all of the selected actions' preconditions

If this fails to find a solution, mark this level and all the goals not satisfied as: (level, goals) (level, goals) stops changing, no solution

## Graph Plan

# Remember this... <br> Initial: $\neg$ Money $\wedge \neg$ Smart $\wedge \neg$ Debt <br> Goal: $\neg$ Money $\wedge$ Smart $\wedge \neg$ Debt 

Action( School,
Precondition:
Effect: Debt $\wedge$ Smart) Effect: Money $\wedge \neg$ Smart) Action( Pay,
Precondition: Money,
Effect: $\neg$ Money $\wedge \neg D e b t)$

Ask:


## Ask:

## ${ }_{7} \mathrm{D}^{\wedge} \wedge^{\wedge} \mathrm{M}$ Graph Plan ${ }_{1 \& 4}$ in mutex



## Ask:

## ${ }_{7} \mathrm{D}^{\wedge} \mathrm{S}_{7} \mathrm{M}$ Graph Plan ${ }_{3 \& 4}$ in mutex

 try different

Ask:


## Finding a solution

Formally, the algorithm is:
graph $=$ initial
noGoods = empty table (hash)
for level $=0$ to infinity
if all goal pairs not in mutex
solution = recursive search with noGoods
if success, return paths
if graph \& noGoods converged, return fail graph = expand graph

Initial: Clean $\wedge$ Garbage $\wedge$ Quiet
Goal: Food $\wedge \neg$ Garbage $\wedge$ Present


