CSci 4271W
Development of Secure Software Systems
Day 19: Public-key Cryptography

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Preview question

Which of the following would have to be completely abandoned if scalable quantum computers become widely available?

A. one-time pads
B. RSA
C. AES
D. ROT-13
E. SHA-3

Outline

Public-key crypto basics

Public key encryption and signatures

Brief introduction to networking

Pre-history of public-key crypto

- First invented in secret at GCHQ
- Proposed by Ralph Merkle for UC Berkeley grad. security class project
  - First attempt only barely practical
  - Professor didn't like it
- Merkle then found more sympathetic Stanford collaborators named Diffie and Hellman

Box and locks analogy

- Alice wants to send Bob a gift in a locked box
  - They don't share a key
  - Can't send key separately, don't trust UPS
  - Box locked by Alice can't be opened by Bob, or vice-versa

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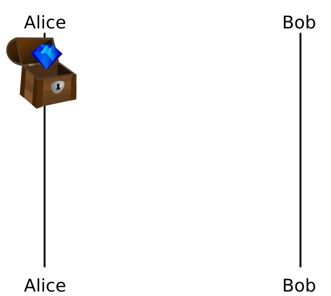
- Math perspective: physical locks commute

Protocol with clip art



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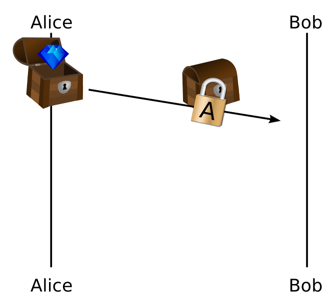

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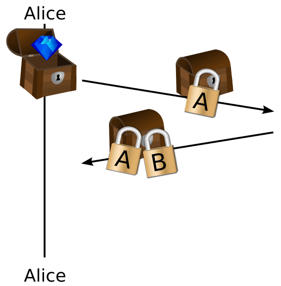

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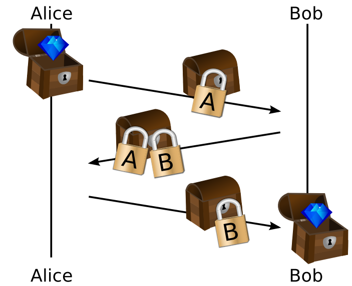

Public key primitives

- Public-key encryption (generalizes block cipher)
  - Separate encryption key EK (public) and decryption key DK (secret)
- Signature scheme (generalizes MAC)
  - Separate signing key SK (secret) and verification key VK (public)

Modular arithmetic

- Fix *modulus* n, keep only remainders mod n
  - mod 12: clock face; mod 2^{32}: `unsigned int`
- $+$, $-$, and $\times$ work mostly the same
- Division? Multiplicative inverse by extended GCD
- Exponentiation: efficient by square and multiply

Generators and discrete log

- Modulo a prime p, non-zero values and $\times$ have a nice ("group") structure
- g is a *generator* if $g^0, g, g^2, g^3, \ldots$ cover all elements
- Easy to compute $x \mapsto g^x$
- Inverse, *discrete logarithm*, hard for large p

Diffie-Hellman key exchange

- Goal: anonymous key exchange
- Public parameters p, g; Alice and Bob have resp. secrets a, b
- Alice→Bob: $A = g^a \pmod{p}$
- Bob→Alice: $B = g^b \pmod{p}$
- Alice computes $B^a = g^{ba} = k$
- Bob computes $A^b = g^{ab} = k$

Relationship to a hard problem

- We're not sure discrete log is hard (likely not even NP-complete), but it's been unsolved for a long time
- If discrete log is easy (e.g., in P), DH is insecure
- Converse might not be true: DH might have other problems

Categorizing assumptions

- Math assumptions unavoidable, but can categorize
- E.g., build more complex scheme, shows it's "as secure" as DH because it has the same underlying assumption
- Commonly "decisional" (DDH) and "computational" (CDH) variants

Key size, elliptic curves

- Need key sizes ~10 times larger then security level
  - Attacks shown up to about 768 bits
- Elliptic curves: objects from higher math with analogous group structure
  - (Only tenuously connected to ellipses)
- Elliptic curve algorithms have smaller keys, about 2× security level

Outline

Public-key crypto basics

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General description

- Public-key encryption (generalizes block cipher)
  - Separate encryption key EK (public) and decryption key DK (secret)
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RSA setup

- Choose $n = pq$, product of two large primes, as modulus
- n is public, but p and q are secret
- Compute encryption and decryption exponents e and d such that

$$M^{ed} = M \pmod{n}$$

RSA encryption

- Public key is (n, e)
- Encryption of M is $C = M^e \pmod{n}$
- Private key is (n, d)
- Decryption of C is $C^d = M^{ed} = M \pmod{n}$

RSA signature

- Signing key is (n, d)
- Signature of M is $S = M^d \pmod{n}$
- Verification key is (n, e)
- Check signature by $S^e = M^{de} = M \pmod{n}$
- Note: symmetry is a nice feature of RSA, not shared by other systems

RSA and factoring

- We're not sure factoring is hard (likely not even NP-complete), but it's been unsolved for a long time
- If factoring is easy (e.g., in P), RSA is insecure
- Converse might not be true: RSA might have other problems

Homomorphism

- Multiply RSA ciphertexts $\Rightarrow$ multiply plaintexts
- This *homomorphism* is useful for some interesting applications
- Even more powerful: fully homomorphic encryption (e.g., both $+$ and $\times$)
  - First demonstrated in 2009; still very inefficient

Problems with vanilla RSA

- Homomorphism leads to chosen-ciphertext attacks
- If message and e are both small compared to n, can compute $M^{1/e}$ over the integers
- Many more complex attacks too

Hybrid encryption

- Public-key operations are slow
- In practice, use them just to set up symmetric session keys

\+ Only pay RSA costs at setup time

— Breaks at either level are fatal

Padding, try #1

- Need to expand message (e.g., AES key) size to match modulus
- PKCS#1 v. 1.5 scheme: prepend 00 01 FF FF .. FF
- Surprising discovery (Bleichenbacher'98): allows adaptive chosen ciphertext attacks on SSL
  - Variants recurred later (c.f. "ROBOT" 2018)

Modern "padding"

- Much more complicated encoding schemes using hashing, random salts, Feistel-like structures, etc.
- Common examples: OAEP for encryption, PSS for signing
- Progress driven largely by improvement in random oracle proofs

Simpler padding alternative

- "Key encapsulation mechanism" (KEM)
- For common case of public-key crypto used for symmetric-key setup
  - Also applies to DH
- Choose RSA message r at random mod n, symmetric key is $H(r)$

— Hard to retrofit, RSA-KEM insecure if e and r reused with different n

Post-quantum cryptography

- One thing quantum computers would be good for is breaking crypto
- Square root speedup of general search
  - Countermeasure: double symmetric security level
- Factoring and discrete log become poly-time
  - DH, RSA, DSA, elliptic curves totally broken
  - Totally new primitives needed (lattices, etc.)
- Not a problem yet, but getting ready

Box and locks revisited

- Alice and Bob's box scheme fails if an intermediary can set up two sets of boxes
  - Middleperson (man-in-the-middle) attack
- Real world analogue: challenges of protocol design and public key distribution

Outline

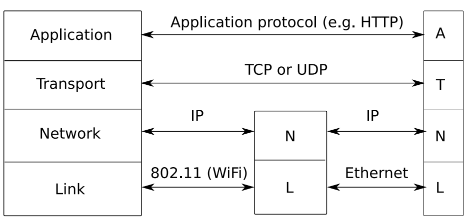
The Internet

- A bunch of computer networks voluntarily interconnected
- Capitalized because there's really only one
- No centralized network-level management
  - But technical collaboration, DNS, etc.

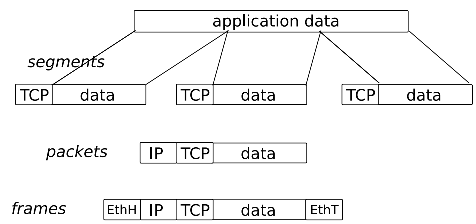
Layered model (OSI)

7. Application (HTTP)
6. Presentation (MIME?)
5. Session (SSL?)
4. Transport (TCP)
3. Network (IP)
2. Data-link (PPP)
1. Physical (10BASE-T)

Layered model: TCP/IP



Packet wrapping



IP(v4) addressing

- Interfaces (hosts or routers) identified by 32-bit addresses
  - Written as four decimal bytes, e.g. 192.168.10.2
- First k bits identify network, $32 - k$ host within network
  - Can't (anymore) tell k from the bits
- We'll run out any year now

IP and ICMP

- Internet Protocol (IP) forwards individual packets
- Packets have source and destination addresses, other options
- Automatic fragmentation (usually avoided)
- ICMP (I Control Message P) adds errors, ping packets, etc.

UDP

- User Datagram Protocol: thin wrapper around IP
- Adds source and destination port numbers (each 16-bit)
- Still connectionless, unreliable
- OK for some small messages

TCP

- Transmission Control Protocol: provides reliable bidirectional stream abstraction
- Packets have sequence numbers, acknowledged in order
- Missed packets resent later

Flow and congestion control

- Flow control: match speed to slowest link
  - "Window" limits number of packets sent but not ACKed
- Congestion control: avoid traffic jams
  - Lost packets signal congestion
  - Additive increase, multiplicative decrease of rate

Routing

- Where do I send this packet next?
  - Table from address ranges to next hops
- Core Internet routers need big tables
- Maintained by complex, insecure, cooperative protocols
  - Internet-level algorithm: BGP (Border Gateway Protocol)

Below IP: ARP

- Address Resolution Protocol maps IP addresses to lower-level address
  - E.g., 48-bit Ethernet MAC address
- Based on local-network broadcast packets
- Complex Ethernets also need their own routing (but called switches)

DNS

- Domain Name System: map more memorable and stable string names to IP addresses
- Hierarchically administered namespace
  - Like Unix paths, but backwards
- `.edu` server delegates to `.umn.edu` server, etc.

DNS caching and reverse DNS

- To be practical, DNS requires caching
  - Of positive and negative results
- But, cache lifetime limited for freshness
- Also, reverse IP to name mapping
  - Based on special top-level domain, IP address written backwards

Classic application: remote login

- Killer app of early Internet: access supercomputers at another university
- Telnet: works cross-OS
  - Send character stream, run regular login program
- rlogin: BSD Unix
  - Can authenticate based on trusting computer connection comes from
  - (Also rsh, rcp)