Implicit restarts
Idea: mix Arnoldi with the QR algorithm in order to apply a filter.. In first case filter $t-\theta$

we have done m steps of Arnoldi – result:

$$A V_m = V_m H_m + \hat{v} e_m^T$$

$$(A - \theta I) V_m = V_m (H_m - \theta I) + \hat{v} e_m^T$$

$(H_m - \theta I) = QR$ all matrice m x m

$(A - \theta I) V_m = V_m Q R + \hat{v} e_m^T$ → multiply by Q to the right

$$(A - \theta I) V_m Q = V_m Q (RQ) + \hat{v} e_m^T Q$$

$H = QR$ then $H^1 = RQ$ ==> one step of the QR algorithm
shift back [add θ I]

$$\underbrace{A V_m Q}_{V_m^1} = V_m Q \underbrace{(RQ + \theta I)}_{H_m^1} + \hat{v} \underbrace{e_m^T Q}_{b_m^T}$$

$$A V_m^1 = V_m^1 H_m^1 + \hat{v} b_m^T$$

H_m^1 = Hessenberg

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Jacobi-Davidson

We want:

$$A (z + v) = (\mu + \eta)(z+v) \rightarrow M (z + v) = (\mu + \eta)(z+v)$$

$(M-\mu I) v - \eta z = - (M z - \mu z) + \eta v$
=== r ====   === small

solve:

$$(M-\mu I)v - \eta z = - r$$

$$w^H v = 0$$

system of (n+1)x(n+1) equations.

$$v = -(M-\mu I)^{-1} r + \eta (M-\mu I)^{-1} z$$